22M:132: Topology Exam 1
Sept 30, 2008

[10] 1.) Definition: The point $x \in X$ is a limit point of A if

[12] 2.) Suppose $\mathbf{R}$ = the set of real numbers has the standard topology. Let $\mathbf{Q}$ = the set of rational numbers. Calculate the following in $\mathbf{R}$:

$\mathbf{Q}^o$ = __________ $\overline{\mathbf{Q}}$ = __________ $\mathbf{Q}'$ = __________

3.) The following two statements are false. Show that the statements are false by providing counter-examples. You do not need to explain your counter-examples.

[9] 3a.) If $x_n \in A$, then there exists a unique point $x \in \overline{A}$ such that $x_n \to x$.

[9] 3b.) If $f : X \to Y$ is continuous, then $\overline{f(A)} \subset f(\overline{A})$.

[60] Prove 2 of the following 3. Clearly indicate your choices.

1. The product of two Hausdorff spaces is Hausdorff.

2. Let Y be a subspace of X. If $A \subset Y$, then $Cl_Y(A) = \overline{A} \cap Y$.

3. Show that $D((x_1, y_1), (x_2, y_2)) = max\{|x_1 - y_1|, \frac{|x_2 - y_2|}{2}\}$ is a metric on $\mathbf{R}^2$ where $(x_i, y_i) \in \mathbf{R}^2$. Show that this metric generates the standard topology on $\mathbf{R}^2$.