Is a compact Hausdorff space locally compact? Yes.
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<u>**Claim.**</u> **All compact spaces are locally compact.**

> **Definition.** A space X is said to be **locally compact at** x if there is some compact subspace C of X that contains a neighborhood of x. If X is locally compact at each of its points, X is said simply to be **locally compact**.   *-Munkres, 2nd Ed., §29*

Since X is a compact space, choose $C := X$. The same choice holds for each $x \in X$, giving us that X is locally compact.