Mathematics Colloquium Thursday 3:30pm in 114 MLH
Mike Williams, UCSB: 3-Manifolds and surface decompositions

26. Compact Sets (continued)

Defn: A collection C is said to have the **finite intersection property** if for every finite subcollection $\{C_1, ..., C_n\} \subset \mathcal{C}$, $\cap_{i=1}^n C_i \neq \emptyset$.

Example 1: $\{(-n, n) \mid n = 1, 2, 3, ...\}$ has/does not have finite intersection property.

Example 2: $\{(n, n+2) \mid n \in \mathcal{Z}\}$ has/does not have finite intersection property.

Example 3: $\{(0, \frac{1}{n}) \mid n = 1, 2, 3, ...\}$ has/does not have finite intersection property.

Thm 26.9: X is compact if and only if for every collection $\mathcal{C}$ of closed sets in X having the finite intersection property, $\cap_{C \in \mathcal{C}} C \neq \emptyset$.

27: Compact subspaces of the real line.

Thm 27.1: Let X be a simply ordered set with th
property. If X has the order topology, then $[a, b]$ is
pact.

Cor: $[a, b] \subset \mathbf{R}$ is compact. $\Pi_{i=1}^n [a_i, b_i] \subset \mathbf{R}^n$ is com

Thm 27.3: A subspace A of R^n (with standard topo
is compact if and only if it is closed and bounded i
euclidean or square metric.