

Mathematics Colloquium Thursday 3:30pm in 114 MLH  
Mike Williams, UCSB: 3-Manifolds and surface decompositions

## 26. Compact Sets (continued)

Defn: A collection  $\mathcal{C}$  is said to have the **finite intersection property** if for every finite subcollection  $\{C_1, \dots, C_n\} \subset \mathcal{C}$ ,  $\bigcap_{i=1}^n C_i \neq \emptyset$ .

Example 1:  $\{(-n, n) \mid n = 1, 2, 3, \dots\}$  has/does not have finite intersection property.

Example 2:  $\{(n, n+2) \mid n \in \mathcal{Z}\}$  has/does not have finite intersection property.

Example 3:  $\{(0, \frac{1}{n}) \mid n = 1, 2, 3, \dots\}$  has/does not have finite intersection property.

Thm 26.9:  $X$  is compact if and only if for every collection  $\mathcal{C}$  of closed sets in  $X$  having the finite intersection property,  $\bigcap_{C \in \mathcal{C}} C \neq \emptyset$ .

## 27: Compact subspaces of the real line.

Thm 27.1: Let  $X$  be a simply ordered set with the least upper bound property. If  $X$  has the order topology, then  $[a, b]$  is compact.

Cor:  $[a, b] \subset \mathbf{R}$  is compact.  $\prod_{i=1}^n [a_i, b_i] \subset \mathbf{R}^n$  is compact.

Thm 27.3: A subspace  $A$  of  $\mathbf{R}^n$  (with standard topology) is compact if and only if it is closed and bounded in the euclidean or square metric.