26. Compact Sets (continued)

Defn: A collection C is said to have the **finite intersection property** if for every finite subcollection $\{C_1, ..., C_n\} \subset \mathcal{C}$, $\cup_{i=1}^{n} C_i \neq \emptyset$.

Example 1: $\{(-n, n) \mid n = 1, 2, 3, ...\}$ has/does not have finite intersection property.

Example 2: $\{(n, n+2) \mid n \in \mathcal{Z}\}$ has/does not have finite intersection property.

Example 3: $\{(0, \frac{1}{n}) \mid n = 1, 2, 3, ...\}$ has/does not have finite intersection property.

Thm 26.9: X is compact if and only if for every collection $\mathcal{C}$ of closed sets in X having the finite intersection property, $\cup_{C \in \mathcal{C}} C \neq \emptyset$.

Thm 27.3: A subspace A of R^n (with standard topology) is compact if and only if it is closed and bounded in the euclidean or square metric.

30. Countability Axioms

Defn: X is said to have a **countable basis at the point** x if there exists a countable collection $\mathcal{B} = \{B_n \mid n \in Z_+\}$ of neighborhoods of x such that if $x \in U^{open}$ implies there exists a $B_i \in \mathcal{B}$ such that $B_i \subset U$.

Defn: X is **first countable** if X has a countable basis at each of its points.

Defn: A space is second countable if it has a countable basis.

Defn: $A \subset X$ is dense in X is $\overline{A} = X$.

31. Separation Axioms

Defn: X is **regular** if one-point sets are closed in X and if for all closed sets B and for all points $x \notin B$, there exist disjoint open sets, U, V, such that $x \in U$ and $B \subset V$.

Defn: X is **normal** if one-point sets are closed in X and if for all pairs of disjoint closed sets A, B, there exist disjoint open sets, U, V, such that $A \subset U$ and $B \subset V$.

Normal implies regular implies Hausdorff implies T_1.

Thm 32.3: Every compact Hausdorff space is normal.

HW (choose 3 - 4): p. 170: 1, 2, 3, 4, 5, p. 199: 8*