

Thm 19.1, 2: Comparison of box and product topologies.
Let \mathcal{B}_α be a basis for X_α

Basis for the box topology: $\{\prod U_\alpha \mid U_\alpha \text{ open in } X_\alpha\}$
or $\{\prod B_\alpha \mid B_\alpha \in \mathcal{B}_\alpha\}$

Basis for the product topology:

$\{\prod U_\alpha \mid U_\alpha \text{ open in } X_\alpha,$
 $U_\alpha = X_\alpha \text{ for all but finitely many } \alpha\}$

or $\{\prod B_\alpha \mid B_{\alpha_i} \in \mathcal{B}_{\alpha_i}, i = 1, \dots, n,$
 $B_\alpha = X_\alpha \text{ for } \alpha \neq \alpha_i, i = 1, \dots, n\}$

Hence box topology is finer than the product topology

Thm 19.3: Let A_α be a subspace of X_α . Then $\prod A_\alpha$ is a subspace of $\prod X_\alpha$ if both products are given the box topology or if both products are given the product topology.

Thm 19.4: If X_α is Hausdorff for all α then $\prod X_\alpha$ is Hausdorff in both the box and product topologies.

HW p. 118: 3, 5, 6, 7

Thm 19.5: $\prod \overline{A_\alpha} = \overline{\prod A_\alpha}$ in both the box and product topologies.

Thm 19.6: Suppose $f_\alpha : X \rightarrow Y_\alpha$. Define $f : X \rightarrow \prod_{\alpha \in A} Y_\alpha$ by $f(x) = (f_\alpha(x))_{\alpha \in A}$. Let $\prod X_\alpha$ have the product topology. Then f is continuous if and only if f_α is continuous $\forall \alpha$