19. The Product Topology.

Defn: Let J be an index set. Given a set X, a **J-tuple** of elements of X is a function $\mathbf{x} : J \to X$. The α**th coordinate of x** $= x_\alpha = \mathbf{x}(\alpha)$.

Defn: Let $\{A_\alpha\}_{\alpha\in J}$ be an indexed family of sets. Let $X = \cup_{\alpha\in J} A_\alpha$. The **Cartesian product** of $\{A_\alpha\}_{\alpha\in J}$, denoted by $\Pi_{\alpha\in J} A_\alpha$, is defined to the the set of all J-tuples $(x_\alpha)_{\alpha\in J}$ of elements of X such that $x_\alpha \in A_\alpha$ for each $\alpha \in J$.

That is, it is the set of all functions $\mathbf{x} : J \to \cup_{\alpha\in J} A_\alpha$ such that $\mathbf{x}(\alpha) \in A_\alpha \forall \alpha \in J$.

Defn: The **box topology** on $\Pi_{\alpha\in J} X_\alpha$ is the topology generated by the basis

$$\{\Pi_{\alpha\in J} U_\alpha \mid U_\alpha \text{ open in } X_\alpha\}.$$

Defn: Let $\mathcal{S}_\alpha = \{\pi_\alpha^{-1}(U) \mid U \text{ open in } X_\alpha\}$
The **product topology** on $\Pi_{\alpha\in J} X_\alpha$ is the topology generated by the subbasis $\mathcal{S} = \cup_{\alpha\in J} \mathcal{S}_\alpha$.