Defn: $x \in X$ is a **limit point** of A iff $x \in U^{open}$ implies $U \cap A - \{x\} \neq \emptyset$.

Defn: A' = the set of all limit points of A.

Thm 17.6: $\overline{A} = A \cup A'$.

Cor 17.7: A closed if and only if $A' \subset A$.

Defn: x_n converges to a limit x if for every neighborhood U of x, there exists a positive integer N such that $n \geq N$ implies $x_n \in U$.

Note: limit point of a set is not the same as limit of a sequence.

Defn: X is **Hausdorff space** if for all $x_1, x_2 \in X$ such that $x_1 \neq x_2$, there exists neighborhoods U_1 and U_2 of x_1 and x_2, respectively, such that $U_1 \cap U_2 = \emptyset$.

Thm 17.8: Every finite point set in a Hausdorff space X is closed.

Defn: X is T_1 if every one point set is closed.

Defn: X is T_1 if $\forall\ x_1, x_2 \in X$ such that $x_1 \neq x_2$, $\exists$ nbhds U_1 and U_2 of x_1 and x_2, respectively, such that $x_2 \notin U_1$ and $x_1 \notin U_2$.

Defn: X is T_1 if $\forall\ x_1 \in X$, $x_2 \neq x_1$ implies $\exists$ a nbhd U of x_1 such that $x_2 \notin U$.

Defn: X is T_0 if $\forall\ x_1, x_2 \in X$ such that $x_1 \neq x_2$, $\exists$ EITHER [a nbhd U of x_1 such that $x_2 \notin U$] or [a nbhd V of x_2 such that $x_1 \notin V$]

Thm 17.9: Let X by T_1, $A \subset X$. Then x is a limit point of A if and only if every neighborhood of x contains infinitely many points of A.

Thm 17.10: If X is Hausdorff, then a sequence of points of X converges to at most one point of X. $\blacksquare$

Thm 17.11: If X has the order topology, then X is Hausdorff. The product of two Hausdorff spaces is Hausdorff. A subspace of a Hausdorff space is Hausdorff.