16. The Subspace Topology.

Defn: Let $(X, \mathcal{T})$ be a topological space, $Y \subset X$. Then the **subspace topology** on Y is the set

$$\mathcal{T}_Y = \{U \cap Y \mid U \in \mathcal{T}\}$$

$(Y, \mathcal{T}_Y)$ is a **subspace** of X.

Lemma 16.1: If $\mathcal{B}$ is a basis for the topology of X, then the set

$$\mathcal{B}_Y = \{B \cap Y \mid B \in \mathcal{B}\}$$

is a basis for the subspace topology on Y.

Lemma 16.2: Let Y be a subspace of X. If U is open in Y and Y is open in X, then U is open in X.

Lemma 16.3: If A_j is a subspace of X_j, $j = 1, 2$, then the product topology on $A_1 \times A_2$ is the same as the topology $A_1 \times A_2$ inherits as a subspace of $X_1 \times X_2$.

Note: Suppose $Y \subset X$ where X is an ordered set with the order topology. The order topology on Y need not be the same as the subspace topology on Y

Ex 1:$(0, 1) \cup \{5\}$

Defn: Suppose $Y \subset X$ where X is an ordered set. Y is **convex** if for all $a, b \in Y$ such that $a < b$, then $(a, b) \subset Y$

Ex. 1: $(1, 2) \cup (3, 4) \subset R$.

Ex. 2: $(1, 2) \cup (3, 4) \subset (1, 2) \cup (3, 9)$.

Lemma 16.4: Let X is an ordered set with the order topology. Let Y be a convex subset of X. Then the order topology on Y is the same as the subspace topology on Y.

HW p91: 1, 3 (prove your answer), 8