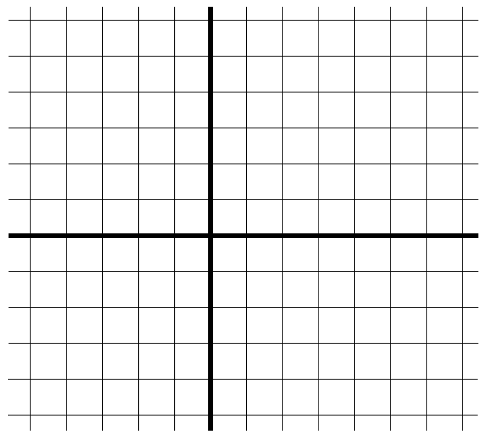
Quiz 2 Section 9*   Sept 27, 2019

[5] 1.) Solve the following first order linear differential equation:

General Solution: ____

[3] 2a.) Sketch the direction field for the autonomous equation. $y' =$



[1] 2b.) On the graph above, sketch the solution with initial value $y(\ \ ) =$

[2] 2c.) Find the equilibrium solutions, and classify them as stable or unstable or semi-stable.

Equilibrium solution: _________.   Stability of this equilibrium solution _____________.

Equilibrium solution: _________.   Stability of this equilibrium solution _____________.

3.) Word problem: State the initial value problem describing ... Do **NOT** solve.

[4] Differential equation: ____

[2] Initial Value: ____

[3] 4.) The solution to the initial value problem $y' = \frac{1-x}{y}$, $y(0) = -\sqrt{3}$ is $y = -\sqrt{-x^2+2x+3}$. State the largest interval on which the solution is defined.

1.) When taking the derivative with respect to t, $(y^2)' = 2y$

A) True   B)False

2.) When taking the derivative with respect to t, $(y^2)' = 2yy'$

A) True   B)False

3.) If $y(0) = -2$ and $y^2 = g(t)$, then $y(t) = \sqrt{g(t)}$

A) True B)False

4.) If $y(0) = -2$ and $y^2 = g(t)$, then $y(t) = -\sqrt{g(t)}$

A) True B)False

We will cover the following next week

1.) If ϕ is a solution to a first order linear differential equation, then $c\phi$ is also a solution to this equation.

A) True B)False

2.) If ϕ is a solution to a first order linear homogeneous differential equation, then $c\phi$ is also a solution to this equation.

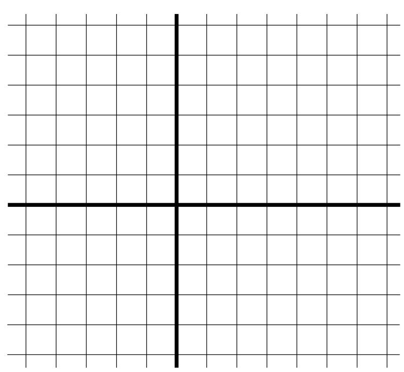
A) True B)False

3.) If ϕ is a solution to a first order linear homogeneous differential equation with constant coefficients, then $c\phi$ is also a solution to this equation.

A) True B)False

From an old quiz:

1. Sketch the direction field for the autonomous equation $y' = y^2 + 2y - 8$. Find the equilibrium solutions, and classify them as stable or unstable. Sketch the solution with initial value $y(0) = 1$.



[2] Equilibrium solution: ______. Stability of this equilibrium solution _________.

[2] Equilibrium solution: ______. Stability of this equilibrium solution _________.

[2] If $y = \phi(t)$ is the solution to the initial value problem $y' = y^2 + 2y - 8$, $y(0) = 3$, what happens to $y = \phi(t)$ as t goes to $+\infty$?

[2] If $y = \phi(t)$ is the solution to the initial value problem $y' = y^2 + 2y - 8$, $y(0) = 1$, what happens to $y = \phi(t)$ as t goes to $+\infty$?

Answer to problem from old quiz:

1. Sketch the direction field for the autonomous equation $y' = y^2 + 2y - 8$. Find the equilibrium solutions, and classify them as stable or unstable. Sketch the solution with initial value $y(0) = 1$.

$y^2 + 2y - 8 = (y + 4)(y - 2) = 0$ implies $y = -4$ and $y = 2$ are equil solns.

[2] Equilibrium solution: $\underline{\ y = -4\ }$. Stability of this equilibrium solution $\underline{\ \textit{stable}\ }$.

[2] Equilibrium solution: $\underline{\ y = 2\ }$. Stability of this equilibrium solution $\underline{\ \textit{unstable}\ }$.

Note the equilibrium solution is a constant solution, not a number. Thus I took off 0.5pt per problem if you did not include $y =$ (i.e, 2 is incorrect, but the equation $y = 2$ is correct).

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[2] If $y = \phi(t)$ is the solution to the initial value problem $y' = y^2 + 2y - 8$, $y(0) = 3$, what happens to $y = \phi(t)$ as t goes to $+\infty$?

Note $y(0) = 3 > 2$. Thus $y \to +\infty$ as $t \to +\infty$

[2] If $y = \phi(t)$ is the solution to the initial value problem $y' = y^2 + 2y - 8$, $y(0) = 1$, what happens to $y = \phi(t)$ as t goes to $+\infty$?

Note $y(0) = 1 \in (-4, 2)$. Thus $y \to -4$ as $t \to +\infty$

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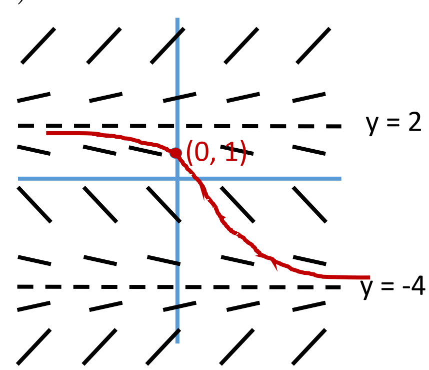
[4] Sketch of Direction field and solution with initial value $y(0) = 1$:



Note one really needs to draw slopes at 9 different y-values (Two for $y > 4$, three for $-2 < y < 4$, and two for $y < -2$ plus the 0-slopes at $y = 4$ and $y = -2$) in order to see all possible solutions.

Note, you don't need to calculate the slopes by plugging in numbers. You just need to know where slope is positive vs negative, small vs large.