TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\frac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\frac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. $t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 31
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 31
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Ex. 7
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a \rvert$	Sec. 6.1; Prob. 8
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a \rvert$	Sec. 6.1; Prob. 7
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 25
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2; Cor. 6.2.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 29

To calculate the **inverse LaPlace transform**, you will use the following algebra techniques:

partial fractions,
completing the square,
adding 0, and
multiplying by 1.

1. Look at denominator

i.) Does your denominator equal one of the following?

$$s^n, \quad s-a, \quad s^2 \pm a^2, \quad (s-a)^{n+1}$$

If so, use the appropriate formula.

ii.) Can you factor denominator over the reals? If so, factor and use partial fractions.

iii.) Do you need to complete the square? Example:

$$\begin{aligned} 10s^2 + 60s + 91 &= 10(s^2 + 6s) + 91 \\ &= 10(s^2 + 6 + 9 - 9) + 91 \\ &= 10(s^2 + 6 + 9) - 90 + 91 \\ &= 10(s+3)^2 + 1 \end{aligned}$$

2. Look at the numerator

i.) Do you need $s - a$? Try adding 0. For example to make $s + \frac{3}{2}$ appear in $5s + 21$:

$$5s + 21 = 5(s + \tfrac{3}{2}) - \tfrac{15}{2} + 21 = 5(s + \tfrac{3}{2}) + \tfrac{27}{2}$$

ii) Do you need b? Try multiplying by 1. For example, if you need $\sqrt{\frac{7}{4}}$, but you have $\frac{27}{2}$: $\quad \frac{27}{2} = \frac{27}{2}\sqrt{\frac{4}{7}}\sqrt{\frac{7}{4}}$