$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ ⟵ IVP soln

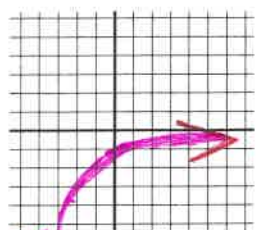
Example 1: Given that the solution to to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

$\begin{bmatrix} -1 \\ 3 \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} \cancel{e^0} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cancel{e^0} \Rightarrow \boxed{c_1 = 1, \quad c_2 = 0}$

$= 1$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

t, x_1-plane



$x_1 = -e^{-2t}$

As $t \to +\infty$

$x_1 \to 0$

$x_2 \to 0$

t, x_2-plane

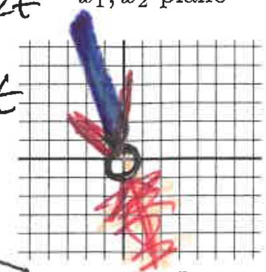
$x_2 = 3e^{-2t}$

$\frac{x_2}{x_1} = \frac{3\cancel{e^{-2t}}}{-\cancel{e^{-2t}}}$

$\frac{x_2}{x_1} = \frac{3}{-1}$

$x_2 = \frac{3}{-1} x_1$

x_1, x_2-plane



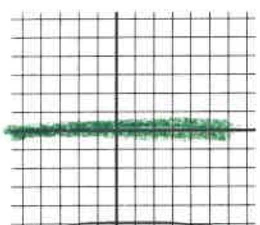
$x_1 < 0$

$x_2 > 0$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

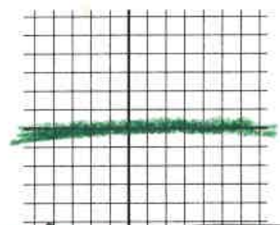
$\Rightarrow c_1 = 0, c_2 = 0$

t, x_1-plane



$x_1 = 0$

t, x_2-plane



$x_2 = 0$

x_1, x_2-plane

$x_1 = 0$

$x_2 = 0$

The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\frac{dx_2}{dx_1} =$ ____________

Plot several direction vectors where the slope is 0 and where slope is vertical.

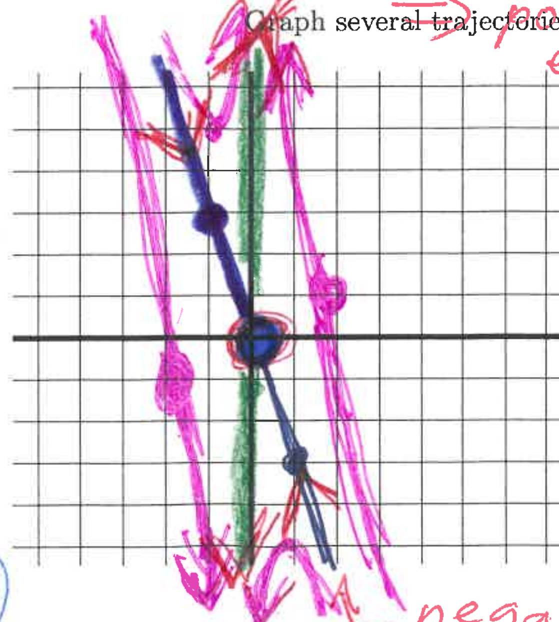
as $t \to +\infty$

soln $\Rightarrow$

$c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

Graph several trajectories.

positive e.value ⟶ positive exponent



negative exponential ⟸ negative eigenvalue

As $t \to +\infty$

$\begin{bmatrix} \cancel{x_1} \\ \cancel{x_2} \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

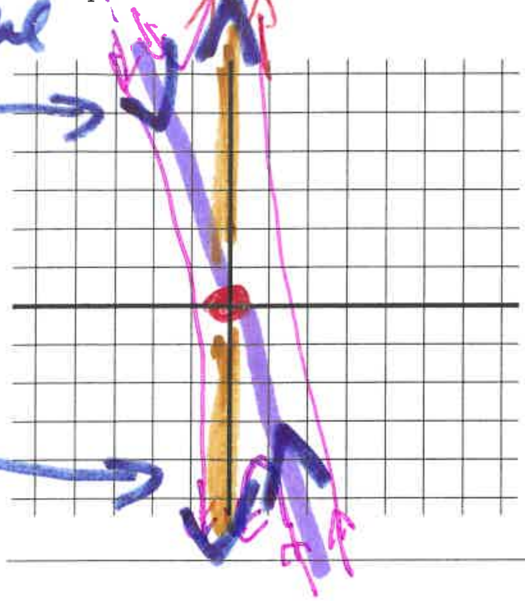
0

7.5 Two real eigenvectors: Graph several trajectories for the following systems of equations:

$\frac{1}{0}$

Example 1: Given that the solution to to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

e. value
-2
+5



$\vec{x}' = A\vec{x}$

The equilibrium solution for this system of equations is

$\Rightarrow \vec{x} = \vec{0}$ is an equil soln

This equilibrium solution is (choose one):

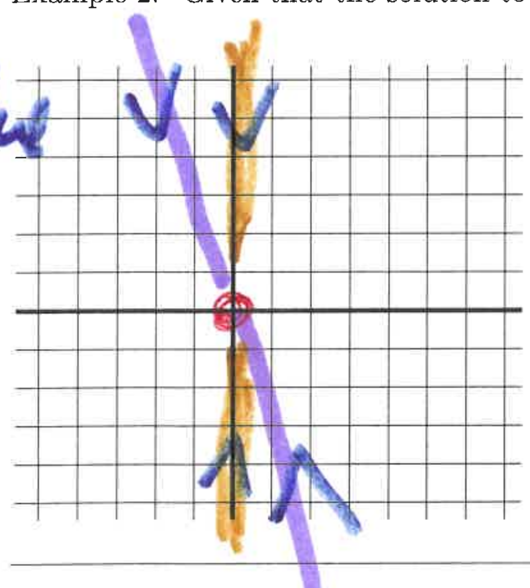
asymptotically stable or unstable

Classify this critical point's type:

$\frac{3}{-1}$

Example 2: Given that the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

e. value
-5
-2



The equilibrium solution for this system of equations is

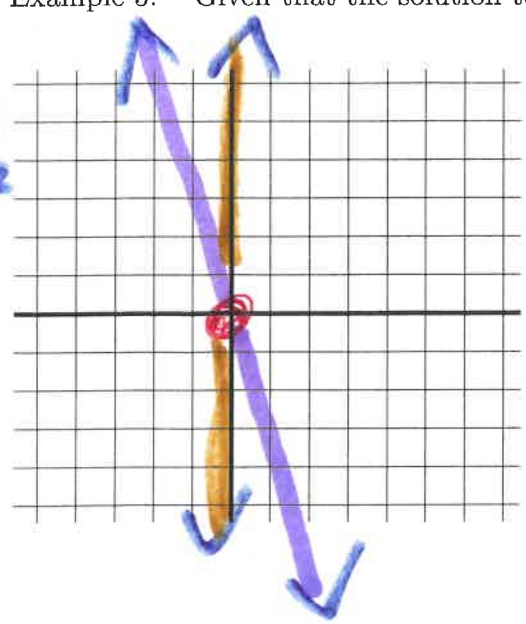
This equilibrium solution is (choose one):

asymptotically stable or unstable

Classify this critical point's type:

Example 3: Given that the solution to $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$

e. value
+5
+2



The equilibrium solution for this system of equations is

This equilibrium solution is (choose one):

asymptotically stable or unstable

Classify this critical point's type:

Example 1: Given that the solution to to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

At $t \to +\infty$, $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} \to 0$ $+ \underbrace{c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}}_{\text{dominates}} \to \text{large}$ approaches $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

If $c_1 \neq 0$ and $c_2 \neq 0$, than for large positive t, the trajectory will be a small perturbation of $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

moving forward in time

At $t \to -\infty$, $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} \to 0$ approaches $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

$t = -100$ $\quad e^{200}$ $\quad e^{-500}$

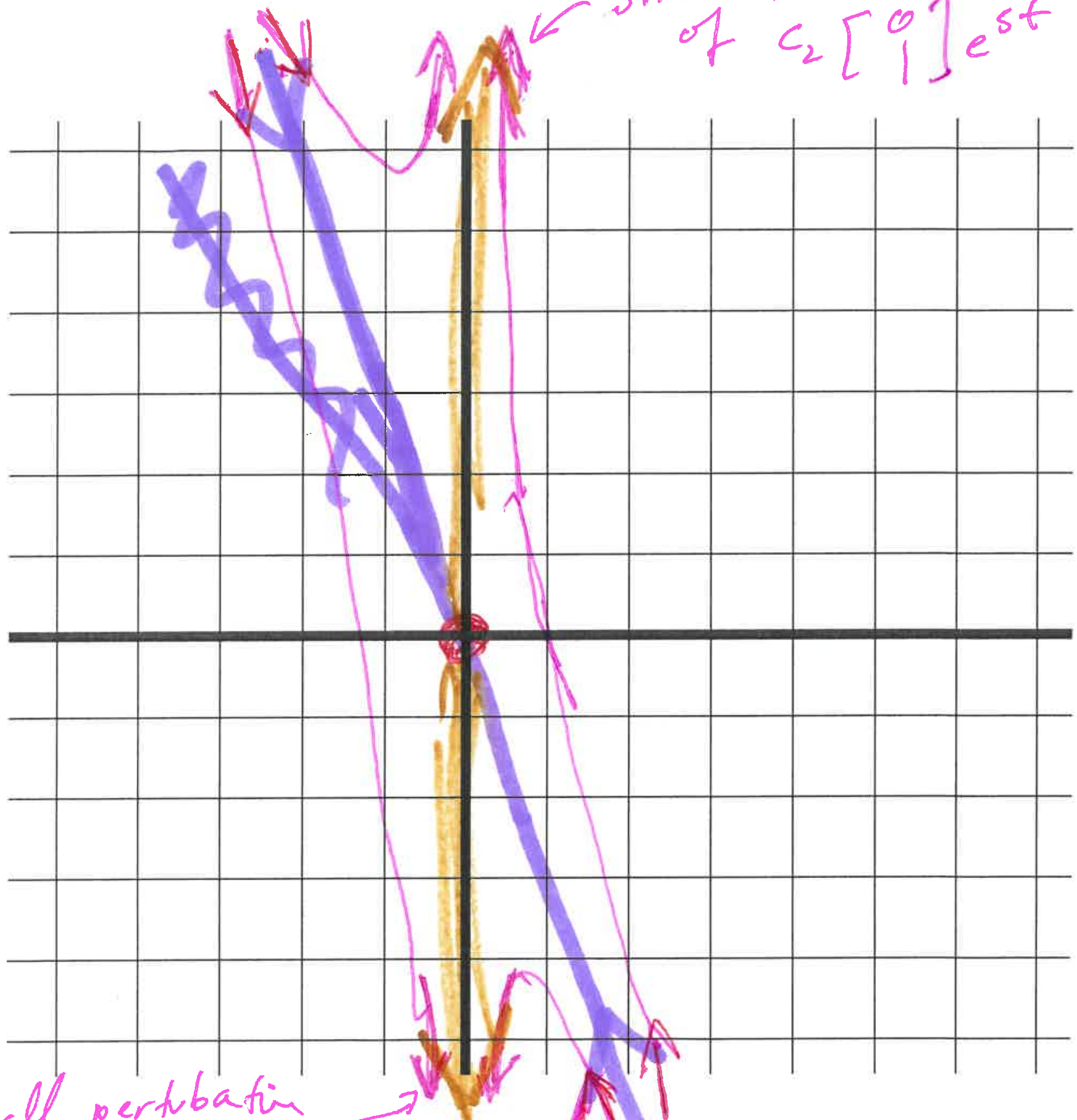
If $c_1 \neq 0$, $c_2 \neq 0$, then for large negative t, the trajectory will be a small perturbation of $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

moving backward in time

7.5 Two real eigenvalues (Example 1: One **positive** and one **negative** eigenvalue).

Example 1: Given that the solution to to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

small pertubation of $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$



small pertubation of $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

The equilibrium solution for this system of equations is $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} =$

This equilibrium solution is (choose one): asymptotically stable or unstable

Classify this critical point's type:

ASSUME $c_1 \neq 0$ and $c_2 \neq 0$

Example 2: Given that the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

At $t \to +\infty$, $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ approaches $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$t = +100 \quad e^{-500} < e^{-200}$

really tiny — small

If $c_1 c_2 \neq 0$, then for large positive t (ie moving forward in time) the trajectory will by a really tiny perturbation of $c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

---

very very large — large

At $t \to -\infty$, $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ approaches $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + \underline{\text{large}}$

$t = -100 \quad e^{500} > e^{200}$

extremely large — large

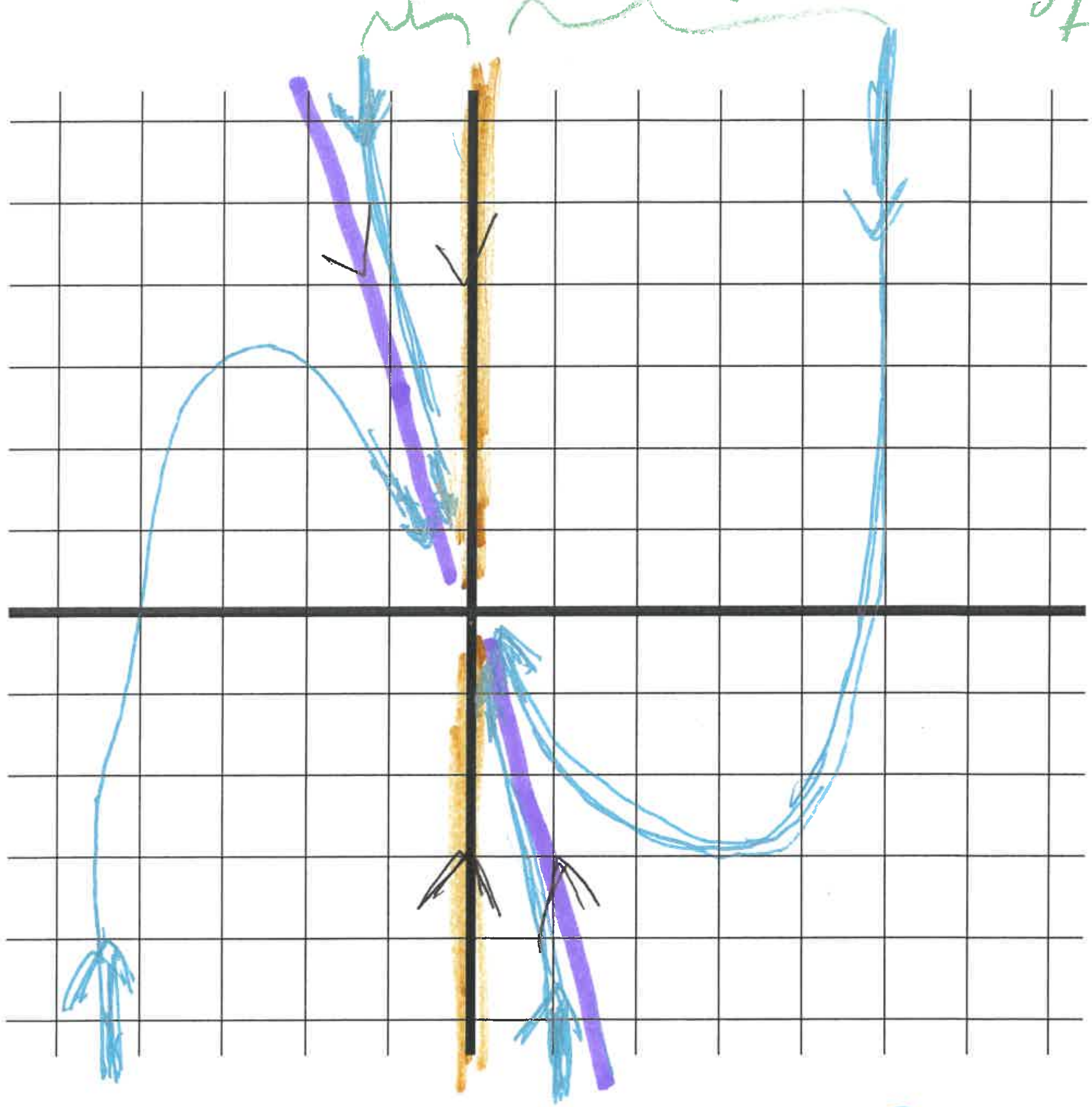
If $c_1 c_2 \neq 0$, then for large negative t (ie moving backwards in time) the trajectory will be a <u>large</u> perturbation of $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t}$

7.5 Two real eigenvalues (Example 3: Two **negative** eigenvalues).

$-5 \qquad -2$

Example 2: Given that the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

large perturbation of 1%



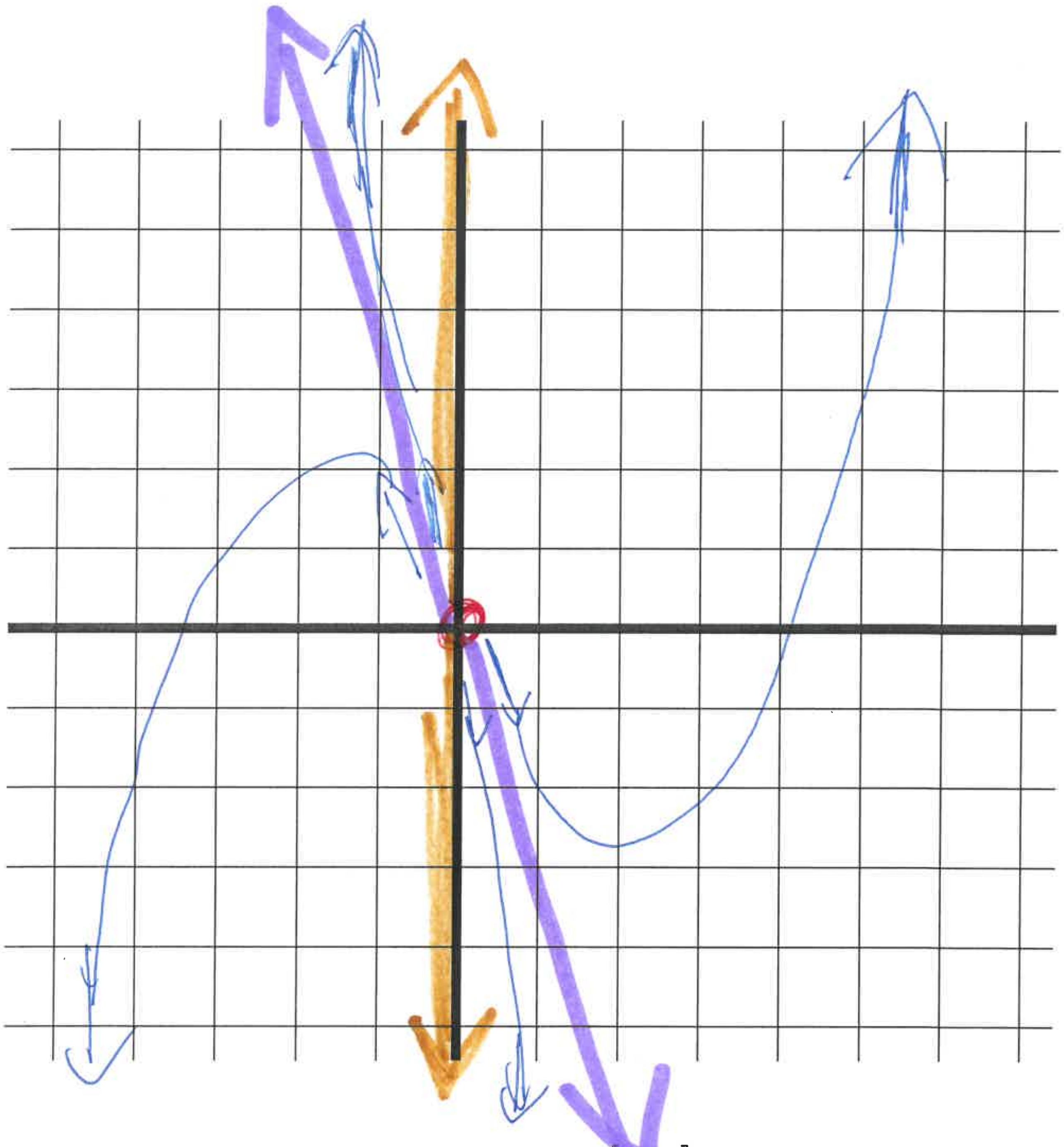
The equilibrium solution for this system of equations is $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This equilibrium solution is (choose one): asymptotically stable or unstable

Classify this critical point's type:

7.5 Two real eigenvalues (Example 2: Two **positive** eigenvalues). +5 +2

Example 3: Given that the solution to $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$



The equilibrium solution for this system of equations is $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} =$

This equilibrium solution is (choose one): asymptotically stable or unstable

Classify this critical point's type:

For $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2x_1 \\ -9x_1 - 5x_2 \end{bmatrix}$

$$x_1' = \frac{dx_1}{dt} = -2x_1$$

$$x_2' = \frac{dx_2}{dt} = -9x_1 - 5x_2$$

$$\frac{dx_2}{dx_1} = \frac{dx_2/dt}{dx_1/dt} = \frac{dx_2}{\cancel{dt}} \cdot \frac{dt}{dx_1} = \frac{-9x_1 - 5x_2}{-2x_1}$$

Slope 0: $-9x_1 - 5x_2 = 0 \Rightarrow x_2 = \frac{+9x_1}{-5} = \frac{+9}{-5} x_1$

Slope ∞: $-2x_1 = 0 \Rightarrow x_1 = 0$

For $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$

$$\frac{dx_1}{dt} =$$

$$\frac{dx_2}{dt} =$$

$$\frac{dx_2}{dx_1} =$$

Slope 0:

Slope ∞: