$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ ← IVP soln

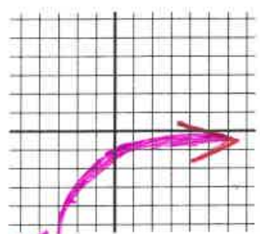
Example 1: Given that the solution to to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix}\mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

$\begin{bmatrix} -1 \\ 3 \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} \cancel{e^0} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cancel{e^0} \Rightarrow \boxed{c_1 = 1 \quad c_2 = 0}$

$=1$

$\boxed{\text{Graph the solution to the IVP } \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \text{ in the}}$

t, x_1-plane



As $t \to +\infty$

$x_1 \to 0$

$x_2 \to 0$

$x_1 = -e^{-2t}$

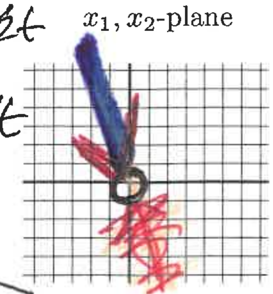
t, x_2-plane

$x_2 = 3e^{-2t}$

$\frac{x_2}{x_1} = \frac{3\cancel{e^{-2t}}}{-\cancel{e^{-2t}}}$

$\frac{x_2}{x_1} = \frac{3}{-1}$

x_1, x_2-plane



$x_1 < 0$

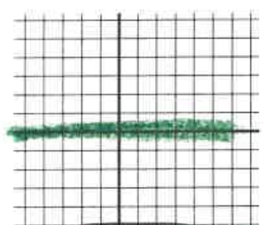
$x_2 > 0$

$x_2 = \frac{3}{-1} x_1$

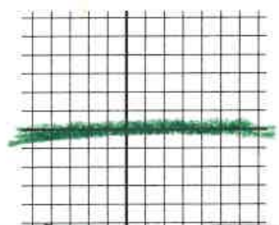
Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

$\Rightarrow c_1 = 0, c_2 = 0$

t, x_1-plane



t, x_2-plane



x_1, x_2-plane

$\boxed{x_1 = 0}$ $\boxed{x_2 = 0}$ $\boxed{x_1 = 0 \quad x_2 = 0}$

The equilibrium solution for this system ~~of equations is~~ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

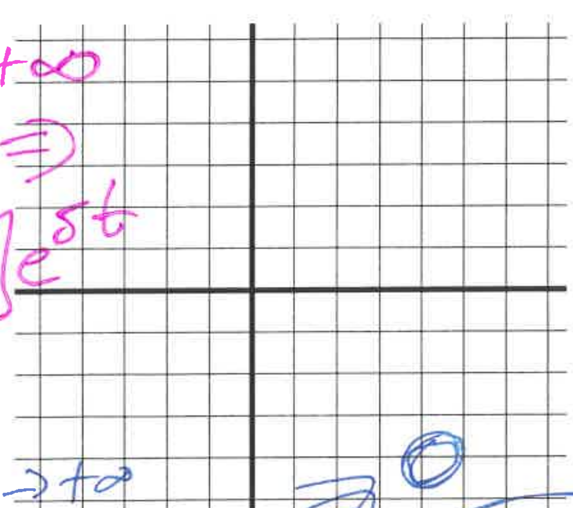
$\frac{dx_2}{dx_1} =$ ______________

Plot several direction vectors where the slope is 0 and where slope is vertical.

as $t \to +\infty$

soln $\Rightarrow$

$c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$



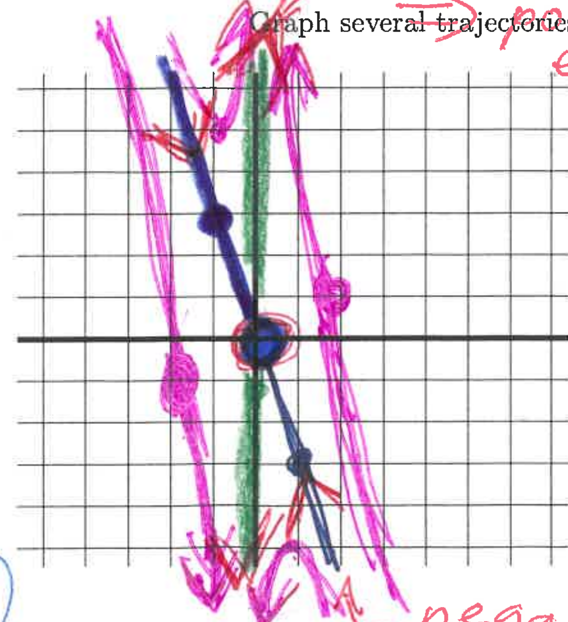
As $t \to +\infty$

$\begin{bmatrix} \cancel{x_1} \\ \cancel{x_2} \end{bmatrix} = c_1 \cancel{\begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

positive e. Val

$\Rightarrow$ positive exponent

Graph several trajectories.



negative eigenvalue

negative exponential $\Leftarrow$

Semi-generic ex: Given that the solution to to $\mathbf{x}' = A\mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ implies $c_1 = 1, c_2 = 0$. Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_1 t}$

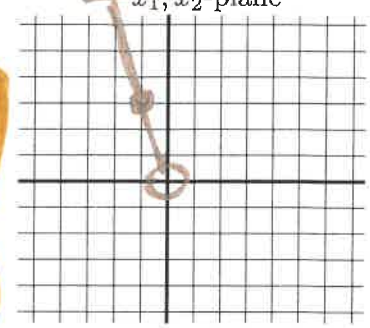
$t = 0$
$x_1 = -1$
$x_2 = 3$

Hence $x_1 = -e^{r_1 t} < 0$ and $x_2 = 3e^{r_1 t} > 0$

and $\frac{x_2}{x_1} = \frac{3e^{r_1 t}}{-e^{r_1 t}} = \frac{3}{-1}$. Thus $x_2 = \frac{3}{-1} x_1$.

https://www.geogebra.org/3d (t, -e∧(-2t), 3*e∧(-2t))

x_1, x_2-plane



II | I
III | IV

→ line segment in quadrant II

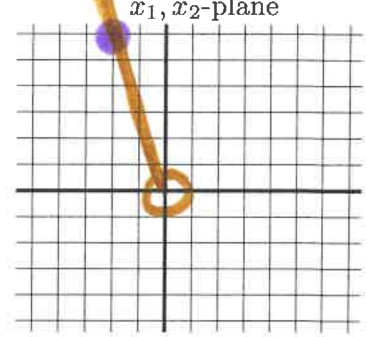
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$ implies $c_1 = 2, c_2 = 0$. Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_1 t}$

Hence $x_1 = -2e^{r_1 t} < 0$ and $x_2 = 6e^{r_1 t} > 0$

and $\frac{x_2}{x_1} = \frac{6e^{r_1 t}}{-2e^{r_1 t}} = \frac{3}{-1}$. Thus $x_2 = \frac{3}{-1} x_1$.

https://www.geogebra.org/3d (t, -2*e∧(-2t), 6*e∧(-2t))

x_1, x_2-plane



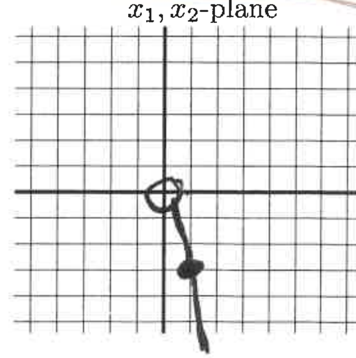
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ implies $c_1 = -1, c_2 = 0$. Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = - \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_1 t}$

Hence $x_1 = e^{r_1 t} > 0$ and $x_2 = -3e^{r_1 t} < 0$

and $\frac{x_2}{x_1} = \frac{3e^{r_1 t}}{-e^{r_1 t}} = \frac{-3}{1}$. Thus $x_2 = \frac{-3}{1} x_1$.

https://www.geogebra.org/3d (t, e∧(-2t), -3*e∧(-2t))

x_1, x_2-plane



→ quadrant IV

$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ is in quadrant IV

Semi-generic ex: Given that the solution to to $\mathbf{x}' = A\mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_2 t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ implies $c_1 = 0, c_2 = 1$.

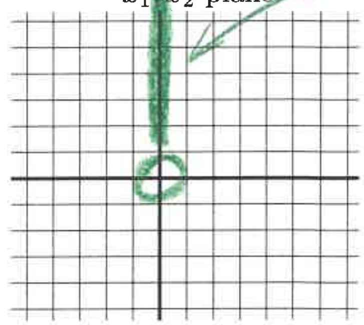
Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

Hence $x_1 = 0$ and $\boxed{x_2 = e^{r_2 t} > 0}$

and $\frac{x_2}{x_1} = \frac{1\cancel{e^{r_2 t}}}{0\cancel{e^{r_2 t}}} = \frac{1}{0}$. Thus $\boxed{x_2 = \frac{1}{0} x_1}$.

https://www.geogebra.org/3d (t, 0, e∧(5t))

x_1, x_2-plane



$x_2 > 0$

IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ implies $c_1 = 0, c_2 = -1$.

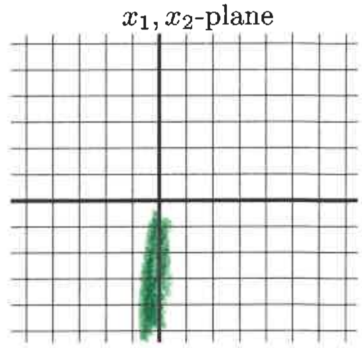
Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -\begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

Hence $x_1 = 0$ and $x_2 = -e^{r_2 t} < 0$

and $\frac{x_2}{x_1} = \frac{-1e^{r_2 t}}{0e^{r_2 t}} = \frac{-1}{0}$. Thus $x_2 = \frac{-1}{0} x_1$.

https://www.geogebra.org/3d (t, 0, -e∧(5t))

x_1, x_2-plane



$x_2 < 0$

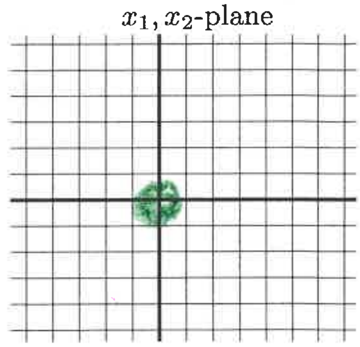
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ implies $c_1 = c_2 = 0$.

Thus IVP soln: $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Hence $x_1 = 0$ and $x_2 = 0$

https://www.geogebra.org/3d (t, 0, 0)

x_1, x_2-plane



$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = c_1 \{ \} + c_2 \{ \}$

$\Rightarrow c_1 = c_2 = 0$

Answer the following questions for $A = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix}$:

when $c_1 = 0$ or $c_2 = 0$ trajectory lies on X

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

For large **positive** values of t which is larger: e^{-2t} or e^{5t}?

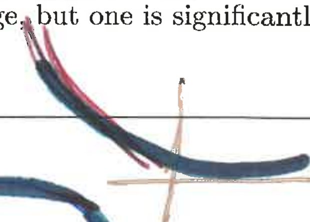
For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$. ← trajectory does NOT lie on X

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ or $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

$t = 100$ $\quad e^{-200} << e^{500}$

* moves away from the origin.
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ 1/0
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ____________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1 t}$ and $||c_2\mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.



For large **negative** values of t which is larger: e^{-2t} or e^{5t}?

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ or $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$?

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior $t \to -\infty$ (select all that apply):

$t = -100$ $\quad e^{200} >> e^{-500}$

* $\lim_{t \to -\infty} \mathbf{x}(t) =$ does not exist.
* $\lim_{t \to -\infty} \mathbf{x}(t) = 0$
* approaches the line $y = mx$ with slope $m =$ 3/-1
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ____________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1 t}$ and $||c_2\mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

Example 3: Given that the solution to $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$

$x_2 = \frac{1}{0} x_1$

$c_1 = 0, c_2 = 1$

$x_1 = -e^{2t}$

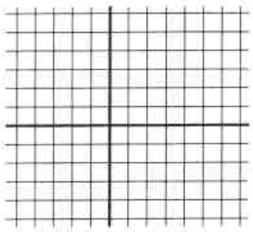
$x_2 = 3e^{2t}$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

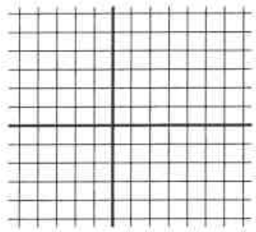
$\frac{x_2}{x_1} = \frac{3e^{2t}}{-e^{2t}}$

$\boxed{x_2 = \frac{3}{-1} x_1}$

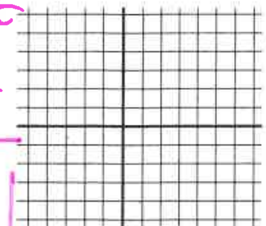
t, x_1-plane



t, x_2-plane

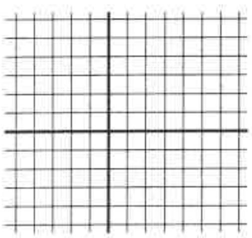


x_1, x_2-plane

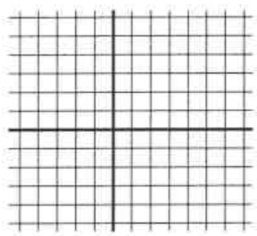


Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

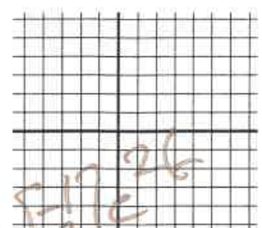
t, x_1-plane



t, x_2-plane



x_1, x_2-plane



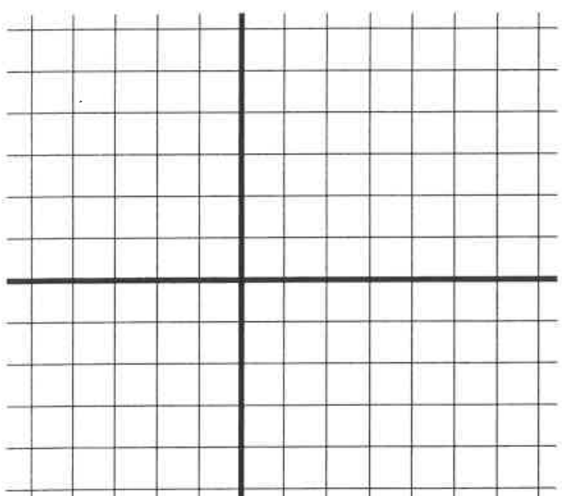
$c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t} + c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

large + dominate

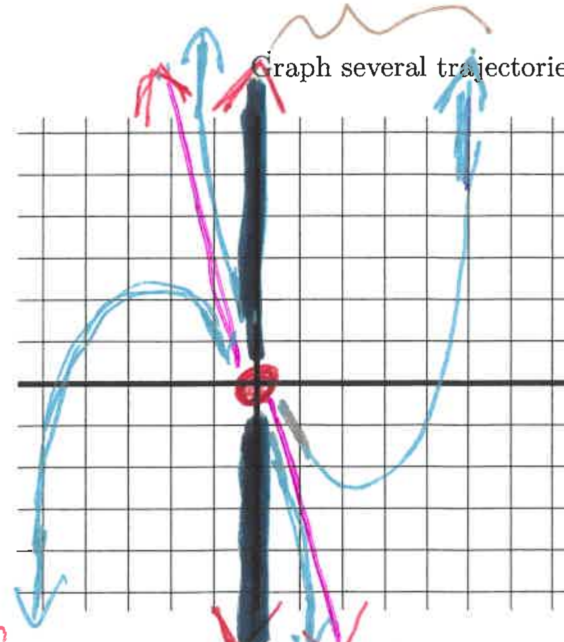
The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\frac{dx_2}{dx_1} =$ ____________

Plot several direction vectors where the slope is 0 and where slope is vertical.



Graph several trajectories.



arrows point away from origin since both e.value positive ⇒ both exponential positive

Answer the following questions for $A = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x} = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

large $\quad$ very very very large

$\vec{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

For large **positive** values of t which is larger: e^{2t} or e^{5t}?

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

For large **positive** values of t, which term dominates: $\quad c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$ or $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

Thus for large **positive** values of t, such trajectories (where $c_1c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

$t = 100$

$e^{200} << e^{500}$

* moves away from the origin.
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ ________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ __$1/0$__. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1t}$ and $||c_2\mathbf{w}||e^{r_2t}$ are large, but one is significantly larger than the other.

large perturbation of $x_2 = \frac{1}{0} x_1$

For large **negative** values of t which is larger: e^{2t} or e^{5t}?

For large **negative** values of t, which term dominates: $\quad c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$ or $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

small perturbat

Thus for large **negative** values of t, such trajectories (where $c_1c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior $t \to -\infty$ (select all that apply):

$t = -100$

$e^{-200} > e^{-500}$

* ~~$\lim_{t \to -\infty} \mathbf{x}(t)$ does not exist.~~
* $\lim_{t \to -\infty} \mathbf{x}(t) = \mathbf{0}$
* approaches the line $y = mx$ with slope $m =$ __$3/-1$__
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1t}$ and $||c_2\mathbf{w}||e^{r_2t}$ are large, but one is significantly larger than the other.

small perturbation of $x_2 = \frac{3}{-1} x_1$

Example 2: Given that the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

$\boxed{X_2 = \frac{1}{0} X}$ $\boxed{X_2 = \frac{3}{-1} X_1}$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

t, x_1-plane t, x_2-plane x_1, x_2-plane

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

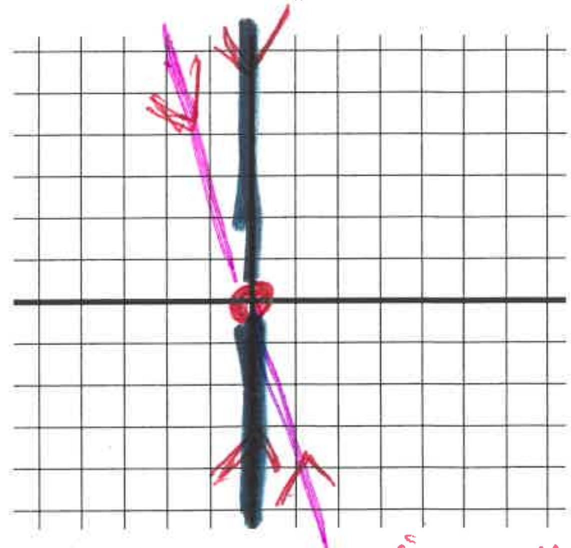
t, x_1-plane t, x_2-plane x_1, x_2-plane

The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\frac{dx_2}{dx_1} =$ ______________

Plot several direction vectors where the slope is 0 and where slope is vertical.

Graph several trajectories.



Two negative e. values $\Rightarrow$ Two negative exponentials $\Rightarrow \vec{0}$ as $t \to +\infty$

Answer the following questions for $A = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix}\mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

For large **positive** values of t which is larger: e^{-5t} or e^{-2t}?

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$,

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t}$ or $c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$?

Thus for large **positive** values of t, such trajectories (where $c_1c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

$t = 100$

$e^{-500} < e^{-200}$

$\frac{1}{e^{500}} < \frac{1}{e^{200}}$

* moves away from the origin.
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ ____________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ____________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1t}$ and $||c_2\mathbf{w}||e^{r_2t}$ are large, but one is significantly larger than the other.

For large **negative** values of t which is larger: e^{-5t} or e^{-2t}?

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t}$ or $c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$?

Thus for large **negative** values of t, such trajectories (where $c_1c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior $t \to -\infty$ (select all that apply):

$t = -100$

$e^{500} \gg e^{200}$

* $\lim_{t \to -\infty} \mathbf{x}(t) =$ does not exist.
* $\lim_{t \to -\infty} \mathbf{x}(t) = \mathbf{0}$
* approaches the line $y = mx$ with slope $m =$ ____________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ____________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1t}$ and $||c_2\mathbf{w}||e^{r_2t}$ are large, but one is significantly larger than the other.