$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} \leftarrow$ IVP soln

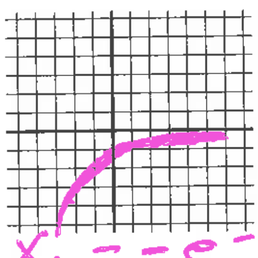
Example 1: Given that the solution to to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

$\begin{bmatrix} -1 \\ 3 \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} \cancel{e^0} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cancel{e^0} \Rightarrow \boxed{\begin{matrix} c_1 = 1 \\ c_2 = 0 \end{matrix}}$

$= 1$

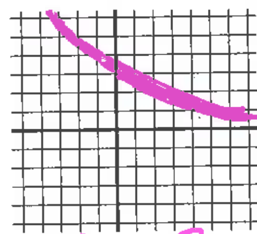
Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

t, x_1-plane



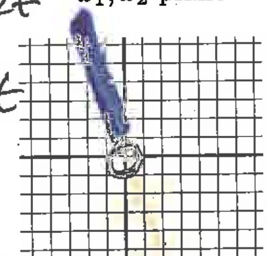
$x_1 = -e^{-2t}$

t, x_2-plane



$x_2 = 3e^{-2t}$

x_1, x_2-plane



$\frac{x_2}{x_1} = \frac{3\cancel{e^{-2t}}}{-\cancel{e^{-2t}}}$

$\frac{x_2}{x_1} = \frac{3}{-1}$

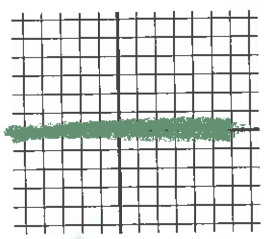
$x_2 = \frac{3}{-1} x_1$

$x_1 < 0$

$x_2 > 0$

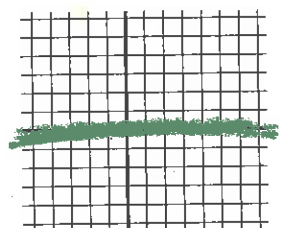
Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

t, x_1-plane



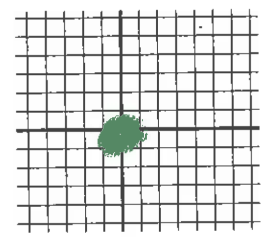
$x_1 = 0$

t, x_2-plane



$x_2 = 0$

x_1, x_2-plane



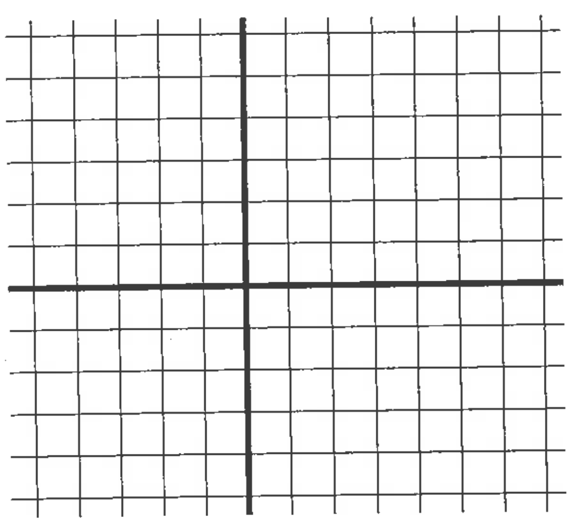
$x_1 = 0$

$x_2 = 0$

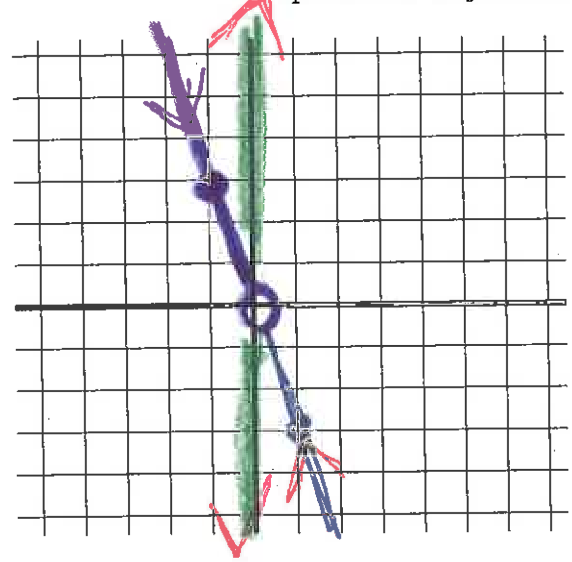
The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\frac{dx_2}{dx_1} =$ ______________

Plot several direction vectors where the slope is 0 and where slope is vertical.



Graph several trajectories.



Semi-generic ex: Given that the solution to to $\mathbf{x}' = A\mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_2 t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ implies $c_1 = 0, c_2 = 1.$

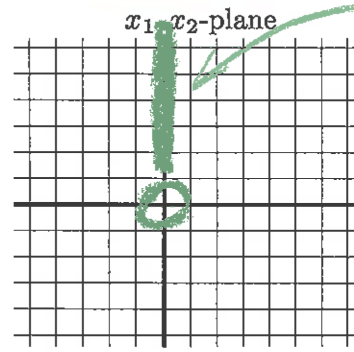
Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

Hence $x_1 = 0$ and $\boxed{x_2 = e^{r_2 t} > 0}$

and $\frac{x_2}{x_1} = \frac{1\cancel{e^{r_2 t}}}{0\cancel{e^{r_2 t}}} = \frac{1}{0}$. Thus $\boxed{x_2 = \frac{1}{0} x_1}$.

https://www.geogebra.org/3d (t, 0, e∧(5t))

$x_2 > 0$



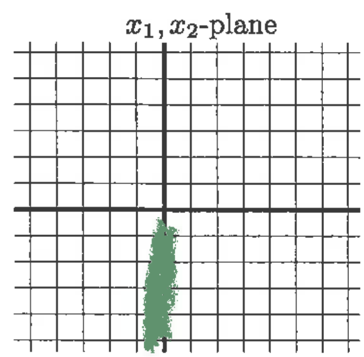
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ implies $c_1 = 0, c_2 = -1.$

Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = - \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

Hence $x_1 = 0$ and $x_2 = -e^{r_2 t} < 0$

and $\frac{x_2}{x_1} = \frac{-1e^{r_2 t}}{0e^{r_2 t}} = \frac{-1}{0}$. Thus $x_2 = \frac{-1}{0} x_1$.

https://www.geogebra.org/3d (t, 0, -e∧(5t))



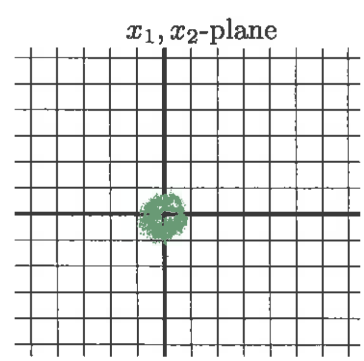
$x_2 < 0$

IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ implies $c_1 = c_2 = 0.$

Thus IVP soln: $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Hence $x_1 = 0$ and $x_2 = 0$

https://www.geogebra.org/3d (t, 0, 0)



$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = c_1 \{\ \} + c_2 \{\ \}$

$\Rightarrow c_1 = c_2 = 0$