Ex: $\underbrace{y' = t + 2y \;,\; y(0) = 0}_{IVP}$

Given $\Delta t = 0.1$

ESTIMATE $y(0.3)$

To find $y(0.3)$ estimate

first estimate $y(0.1)$ and $y(0.2)$

since step size $= \Delta t = 0.1$

---

estimate $y(0.1)$

$y(0.1) = y + \Delta y = 0 + \Delta y = 0 + f'(0,0)(0.1)$

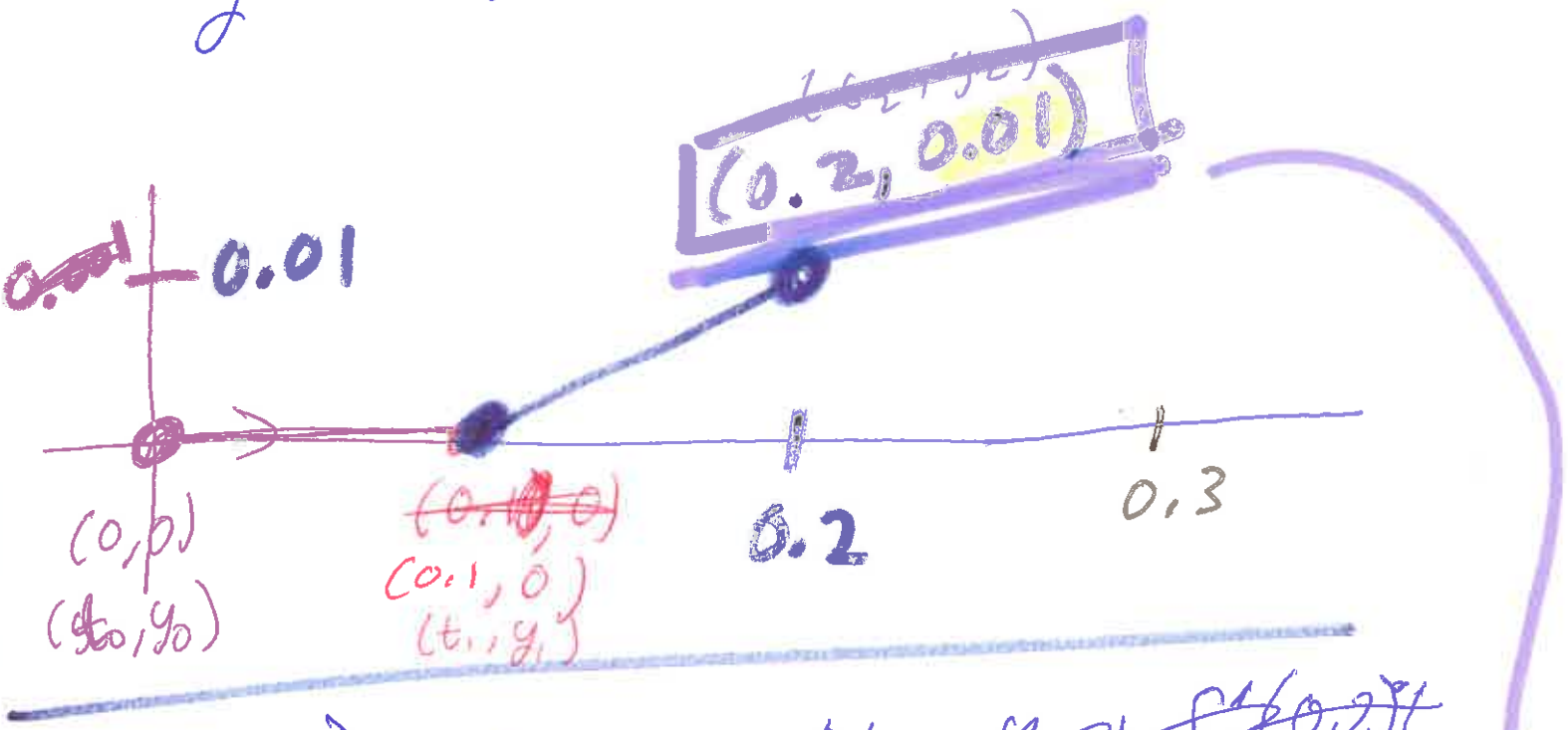
↑ starting at $y = 0$

slope $y' = t + 2y$ when $(t, y) = (0, 0)$

$f'(t, y) = t + 2y$

$f'(0,0) = 0 + 2(0) = 0$

$y(0.2) \approx$ ~~0.00~~ 0.01



$y(0.3) \approx y_2 + \Delta y = y_2 + f'(t_2, y_2)\,\Delta t$

$= 0.01 + f'(0.2, 0.01)(0.1)$

$= 0.01 + f'(0.2, 0.01) \cdot (0.1)$

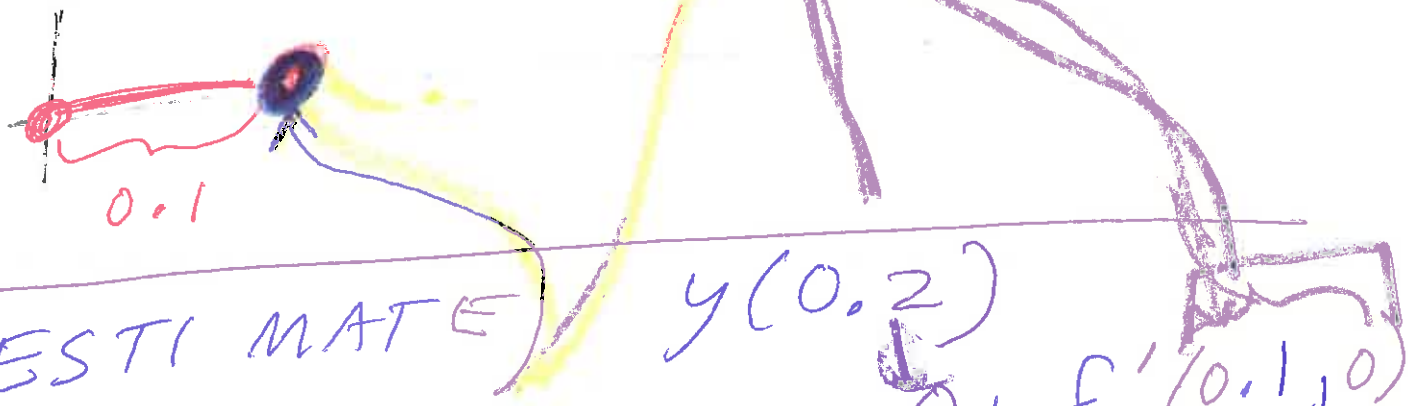
y_2 ; slope at (t_2, y_2) ; Δt

ESTIMATE $y(0.1)$

$y(0.1) = 0 + \Delta y = 0 + f'(0,0) \cdot (0.1)$

$= 0 + 0 \cdot 0.1 = 0$

$(t_1, y_1) = (0.1, 0)$



ESTIMATE $y(0.2)$

$y(0.2) = 0 + \Delta y = 0 + f'(0.1, 0) \cdot (0.1)$

y_1 + slope $\cdot \Delta t$

$= 0 + [0.1 + 2(0)](0.1) = 0.01$

since $f'(t,y) = y' = t + 2y$

$f'(0.1, 0) = 0.1 + 2(0) = 0.1$

$$y(0.3) = y_2 + f'(t_2, y_2) \Delta t$$

$$= (0.01) + f(0.2, 0.01) \cdot (0.1)$$

$$= 0.01 + [0.2 + 2(0.01)] \cdot (0.1)$$

$$= 0.01 + [0.22] \cdot (0.1)$$

$$= 0.01 + 0.0022$$

$$= \boxed{0.032 = y(0.3)}$$