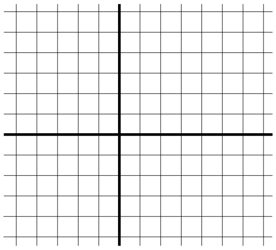
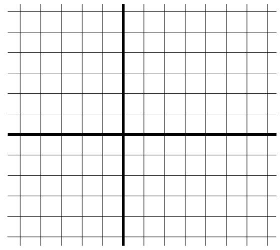
1.) Give that the solution to $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} -2 \\ 3 \end{bmatrix} e^{-2t}$

[4] a.) Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ in the

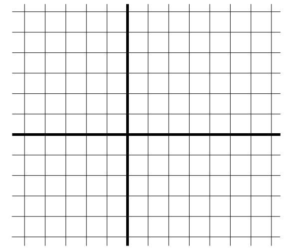
t, x_1-plane



t, x_2-plane

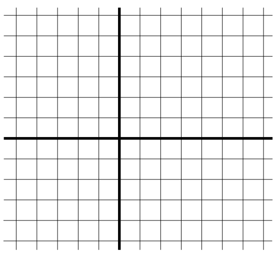


x_1, x_2-plane

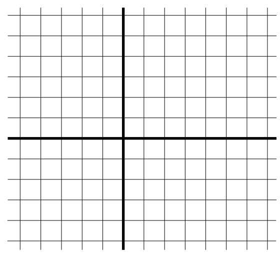


[2] b.) Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

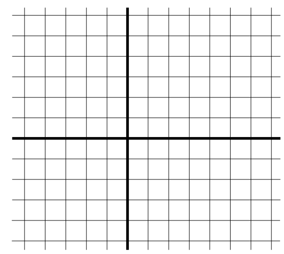
t, x_1-plane



t, x_2-plane



x_1, x_2-plane

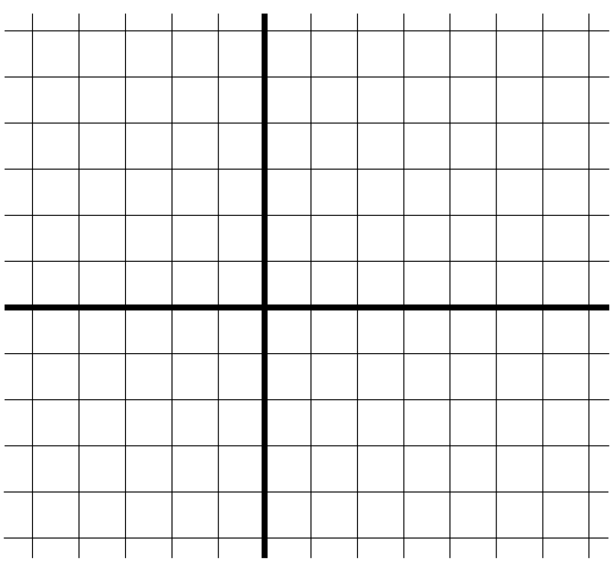


[2] c.) The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$.

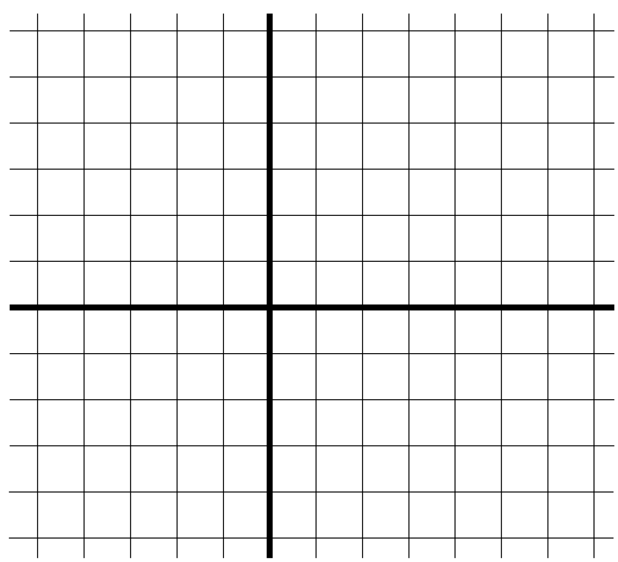
[2] d.) Determine the stability and type of this equilibrium solution:________________________________

[1] e.) $\frac{dx_2}{dx_1}$ = ________________

[9] f.) Graph several trajectories.



extra graph: use only if you wish to



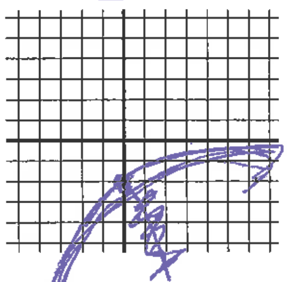
graph for part f

1.) Give that the solution to $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \mathbf{x}$ is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\mathbf{x} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} -2 \\ 3 \end{bmatrix} e^{-2t}$

$c_1 = 0, c_2 = 1 \Rightarrow$ $x_1 = -2e^{-2t}$, $x_2 = 3e^{-2t}$

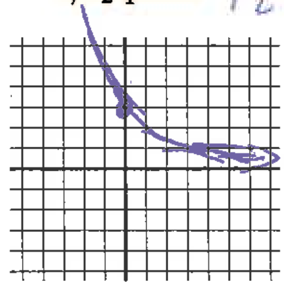
[4] a.) Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ in the

t, x_1-plane $x_1 = -2e^{-2t}$



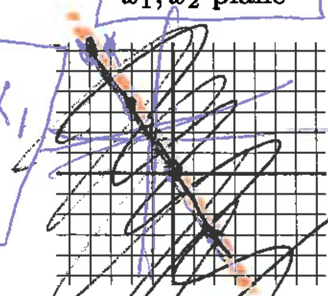
$\frac{x_2}{x_1} = \frac{3e^{-2t}}{-2e^{-2t}}$

t, x_2-plane $x_2 = 3e^{-2t}$



$x_2 = \frac{3}{-2} x_1$

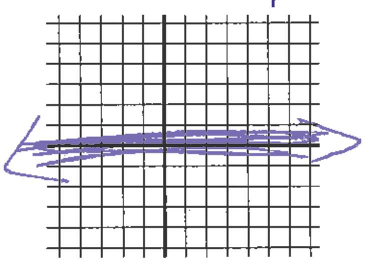
x_1, x_2-plane



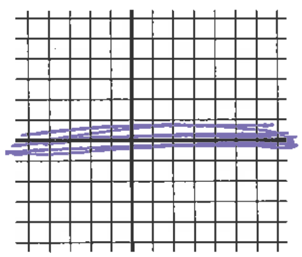
messed up — see graph bottom left

[2] b.) Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the $(c_1 = 0, c_2 = 0) \Rightarrow x_1(t) = 0$, $x_2(t) = 0$

t, x_1-plane $x_1 = 0$

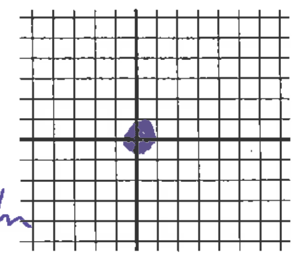


t, x_2-plane $x_2 = 0$



$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is a constant soln

x_1, x_2-plane

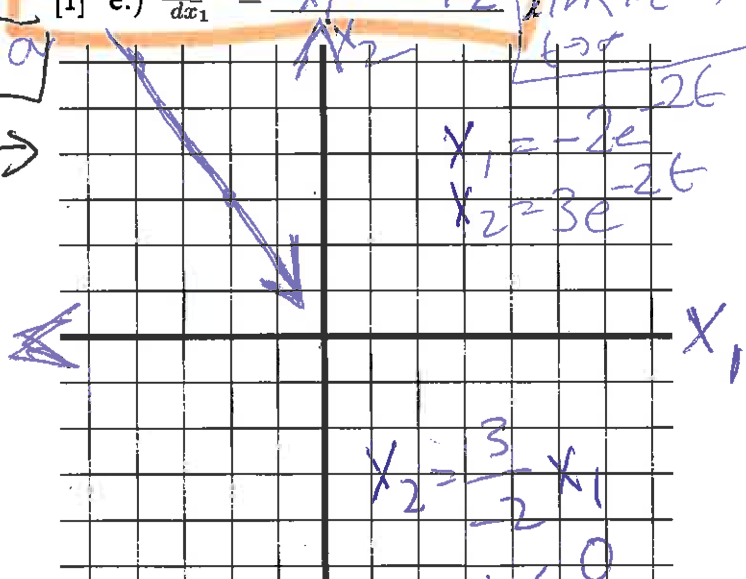


[2] c.) The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. $\vec{x}' = A\vec{x}$

[2] d.) Determine the stability (and type) of the equilibrium solution: unstable saddle

[1] e.) $\frac{dx_2}{dx_1} = \frac{3x_1}{x_1 + 2x_2}$ $\lim_{t \to \infty} x_i(t) = 0$

10



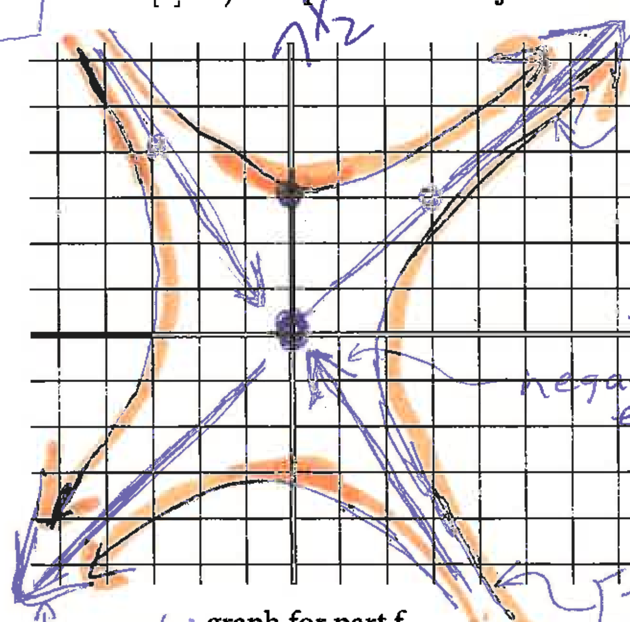
$x_1 = -2e^{-2t}$, $x_2 = 3e^{-2t}$

$x_2 = \frac{3}{-2} x_1$

$x_1 < 0$, $x_2 > 0$

extra graph: use only if you wish to

[9] f.) Graph several trajectories. (include arrows in $x_1 x_2$ plane)



$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $x_2 = \frac{1}{1} x_1$

negative expon

$x_2 = \frac{3}{-2} x_1$ $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

positive exp

graph for part f

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} dx_1/dt \\ dx_2/dt \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ 3x_1 + 0 \end{bmatrix}$$

$$\left.\begin{aligned} \frac{dx_1}{dt} &= x_1 + 2x_2 \\ \frac{dx_2}{dt} &= 3x_1 \end{aligned}\right\} \quad \frac{dx_2/dt}{dx_1/dt} = \frac{dx_2}{\cancel{dt}} \cdot \frac{\cancel{dt}}{dx_1}$$

$$\frac{dx_2}{dx_1} = \frac{3x_1}{x_1 + 2x_2}$$

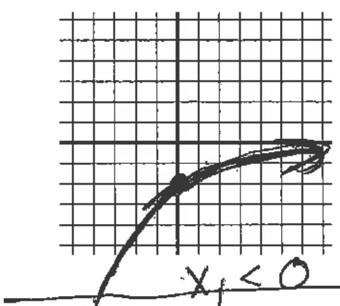
1e

1.) Give that the solution to $\mathbf{x}' = \begin{bmatrix}1 & 2\\ 3 & 0\end{bmatrix}\mathbf{x}$ is $\begin{bmatrix}x_1\\x_2\end{bmatrix} = c_1\begin{bmatrix}1\\1\end{bmatrix}e^{3t} + c_2\begin{bmatrix}-2\\3\end{bmatrix}e^{-2t}$

$c_1 = 0$ & $c_2 = 1 \Rightarrow$ $x_1 = -2e^{-2t}$, $x_2 = 3e^{-2t}$

[4] a.) Graph the solution to the IVP $\begin{bmatrix}x_1(0)\\x_2(0)\end{bmatrix} = \begin{bmatrix}-2\\3\end{bmatrix}$ in the

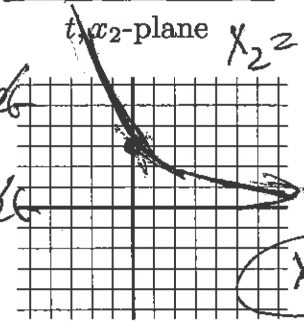
t, x_1-plane $x_1 = -2e^{-2t}$



$x_1 < 0$

$\frac{x_2}{x_1} = \frac{3\cancel{e^{-2t}}}{-2\cancel{e^{-2t}}}$

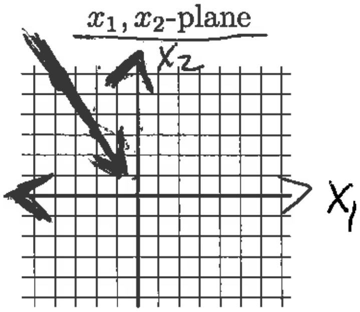
t, x_2-plane $x_2 = 3e^{-2t}$



$x_2 > 0$

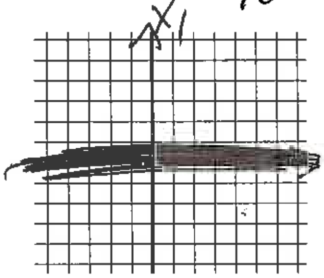
$x_2 = \frac{3}{-2}x_1$

x_1, x_2-plane

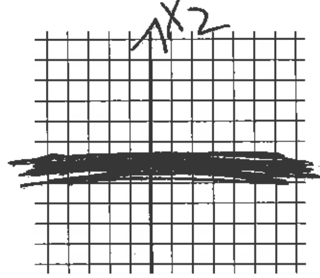


[2] b.) Graph the solution to the IVP $\begin{bmatrix}x_1(0)\\x_2(0)\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ in the $(c_1 = 0$ & $c_2 = 0)$

t, x_1-plane $x_1(t) = 0$



t, x_2-plane

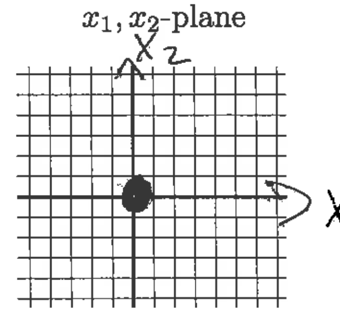


$x_1(t) = 0$

$x_2(t) = 0$

$\vec{x} = 0$ is a constant soln

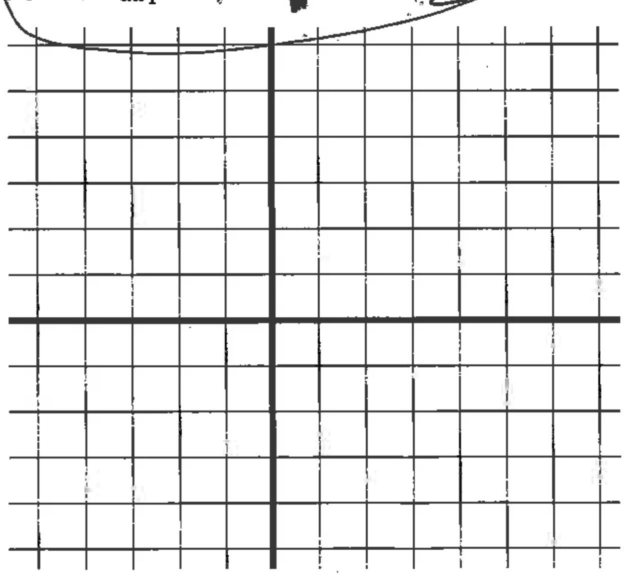
x_1, x_2-plane



[2] c.) The equilibrium solution for this system of equations is $\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$. ← $\vec{x}' = A\vec{x}$

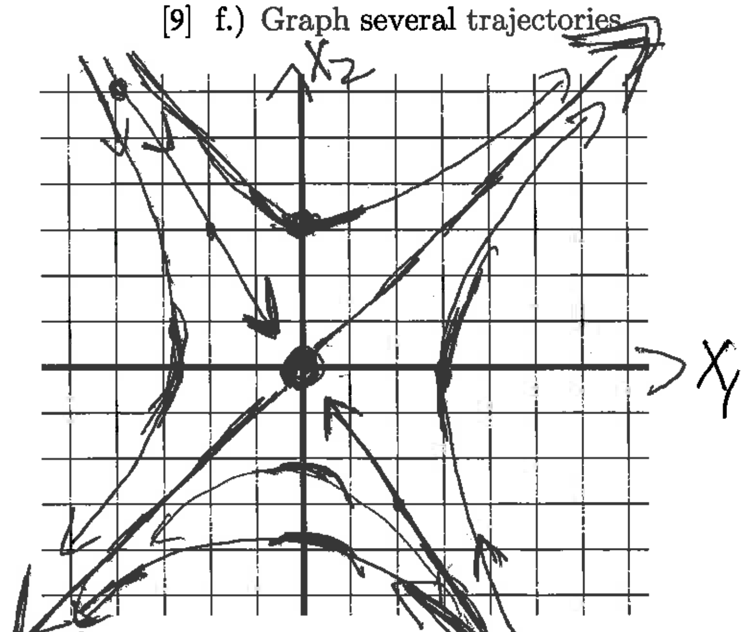
[2] d.) Determine the stability and type of the equilibrium solution: unstable

[1] e.) $\frac{dx_2}{dx_1} = \frac{3x_1}{x_1 + 2x_2}$



extra graph: use only if you wish to

[9] f.) Graph several trajectories



graph for part f

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} dx_1/dt \\ dx_2/dt \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} dx_1/dt \\ dx_2/dt \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ 3x_1 + 0x_2 \end{bmatrix}$$

$$\left.\begin{aligned} \frac{dx_1}{dt} &= x_1 + 2x_2 \\ \frac{dx_2}{dt} &= 3x_1 \end{aligned}\right\} \quad \frac{\frac{dx_2}{\cancel{dt}}}{\frac{dx_1}{\cancel{dt}}} = \frac{dx_2}{\cancel{dt}} \cdot \frac{\cancel{dt}}{dx_1}$$

$$\frac{dx_2}{dx_1} = \frac{3x_1}{x_1 + 2x_2}$$

1e