Solve $\dfrac{dQ}{dt} = 6 - \dfrac{Q}{2}$, $Q(0) = 5$

Turn into calc 1 problem by

§1.2 = §2.2 Separation of variables

$$2\cancel{dt}\left[\frac{dQ}{\cancel{dt}}\right] = \left[6 - \frac{Q}{2}\right]2dt$$

$$\frac{2\,dQ}{12-Q} = \frac{(12-Q)\,dt}{12-Q}$$

$$2\int \frac{dQ}{12-Q} = \int dt$$

$\boxed{\text{(or use u-sub: Let } u = 12-Q,\ du = -dQ)}$

$$-2\ln|12-Q| = t + C$$

Solve for Q:

$$\ln|12-Q| = -\frac{t}{2} + \left(\frac{C}{-2}\right)$$ ← constant

$$\ln|12-Q| = -\frac{t}{2} + C$$ ← sloppy to use same variable but allowed to swallow constants in this class

e $\qquad$ e

$$|12-Q| = e^{-t/2}e^{c} = e^{-t/2}(C)$$ since e^c is a constant

$$12-Q = \pm Ce^{-t/2} = Ce^{-t/2}$$ since $\pm C$ is a constant

$$-(-Q) = -(Ce^{-t/2} - 12)$$

$$\boxed{Q(t) = Ce^{-t/2} + 12}$$ since $-C$ is a constant

Plug in initial value to find C

$Q(0)=5$: $\quad 5 = Ce^{0} + 12$

$$\Rightarrow C = 5-12 = -7$$

$$\Rightarrow \boxed{Q(t) = -7e^{-t/2} + 12 \text{ is soln to IVP}}$$

$$\lim_{t\to\infty}\left(-7e^{-t/2}+12\right) = 0+12 = 12$$

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§1.3 Example: $yy' = 0$ is <u>NOT</u> linear.

SLIDE 6:

Equilibrium sol'ns

$y=2, \quad y=0, \quad y=-1$

Thus these are sol'ns

to $y' = 0 = f(y)$

⇒ Possible DE is

$y' = f(y) = \cancel{(y-2)\,y\,(y+1)}$

But near $y=2$, converges as $t \to \infty$, $y \to 2$

near $y=0$, slope is positive on both sides of $y=0$

near $y=-1$, as $t \to \infty$, y diverges from $t=1$

$y' = -(y-2)(y^2)(y+1)$

$(y-2)$	(y^2)	$(y+1)$	y	slope
$(-)$ $(+)$	$(+)$	$(+)$		$-$
			2	0
$(-)$ $(-)$	$(+)$	$(+)$		$+$
			0	0
$(-)$ $(-)$	$(+)$	$(+)$		$+$
			-1	0
$(-)$ $(-)$	$(+)$	$(-)$		$-$

$y' = -(y-2)y^2(y+1)$