Quizzes are now due on Tuesdays w/ 1 day automatic extension (please finish before Wednesday's class).

Any questions? 2.2 #18, 2.1 #17; 2.1 8a

2.2 #18 $(3y^2-4)\cancel{dx}\left[\frac{dy}{\cancel{dx}}\right] = \left(\frac{3x^2}{\cancel{3y^2-4}}\right)(\cancel{3y^2-4})dx$, $y(1)=0$

Find domain

$$\int (3y^2-4)dy = \int 3x^2\,dx$$

$$\boxed{y^3 - 4y = x^3 + C}$$

implicit general sol'n

IVP: $y(0)=1$

$$1^3 - 4(1) = 0^3 + C$$

$1 - 4(1) = C$

$-3 = C$

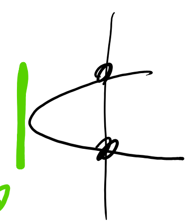
$$\boxed{\text{IVP soln: } y^3 - 4y = x^3 - 3}$$

x indep
y depend $y(x)$

Find interval (x_1, x_2) for which the function y is defined

$$y' = \frac{3x^2}{3y^2 - 4}$$

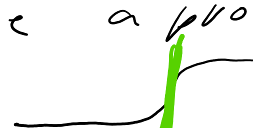
want to avoid



vertical tangent line

not a function

vertical tangent might not be a problem



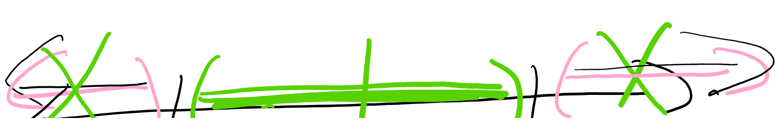
but we will avoid all vertical tangents just in case.

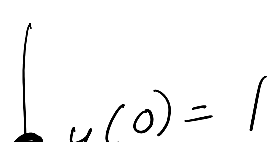
$3y^2 - 4 = 0$

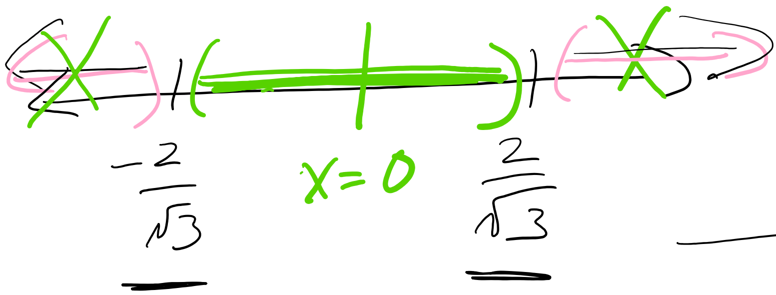
slope ∞

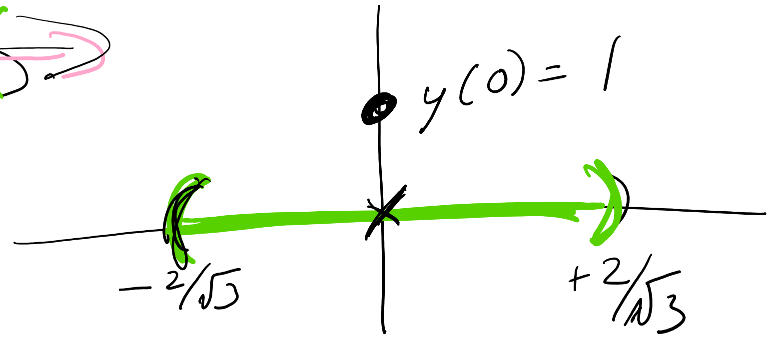
$3y^2 - 4 = 0 \Rightarrow \frac{3}{3}y^2 = 4/3$

$y^2 = \frac{4}{3} \Rightarrow y = \pm \frac{2}{\sqrt{3}}$









$x = 0 \in \left(-\frac{2}{\sqrt3}, \frac{2}{\sqrt3}\right)$

$\Rightarrow$ $\boxed{\text{Domain is } \left(-\frac{2}{\sqrt3}, \frac{2}{\sqrt3}\right)}$

Can ✓ check answer by using software partially to draw direction field

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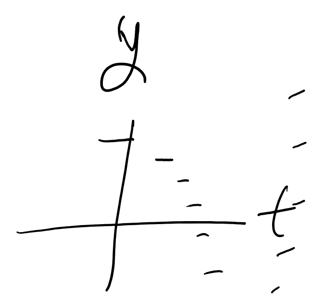
2.1 #17 $\quad y' + \frac{1}{2}y = 2\cos t, \quad y(0) = -1$

Find coordinates of 1st local maximum

Answer: Maximum occurs when $y' = 0$ (or DNE)

$$0 + \frac{1}{2}y = 2\cos t$$

$$\Rightarrow y = 4\cos t$$



Solve IVP: $y' + \frac{1}{2}y = 2\cos t \quad y(0) = 1$

Solve IVP: $y' + \frac{1}{2}y = 2\cos t \quad y(0) = 1$

etc: combine knowing max occurs at some (t, y) where $y = 4\cos t$ w/ sol'n to IVP

2.1.8a

$$2y' + y = 3t^2$$

a) use software to plot

b) note as $t \to \infty$, $y \to \infty$

2.3.3 C from a

$$Q(t) = \frac{63150}{2501} e^{-t/50} + 25$$

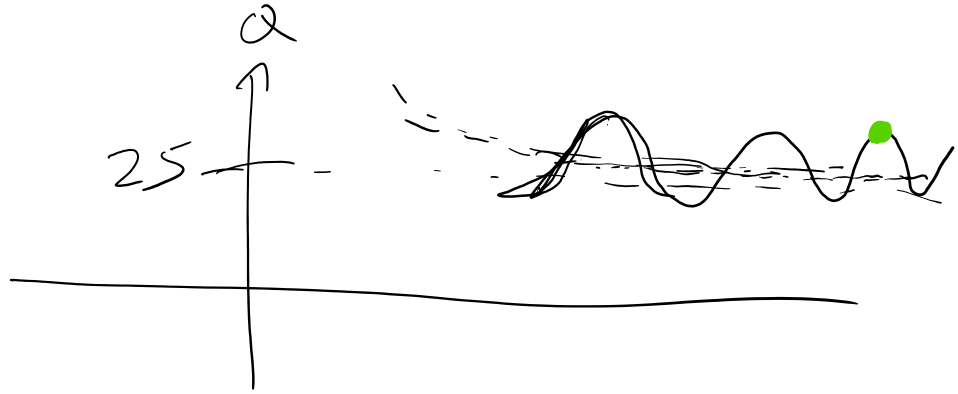
$$- \frac{625}{2501} \cos t + \frac{25}{5002} \sin t$$

for part C, $Q(t)$ oscillates about $Q = 25$ w/ amplitude ____

$$Q(t) = e^{-t/50} + 25 + \underbrace{R\cos(\omega t - \delta)}_{\substack{\text{cosine sum formula}\\ \text{for sum of 2 angles}}}$$

as $t \to \infty$, $e^{-t/50} \to 0$

$$Q(t) \longrightarrow 25 + R\cos(\omega t - \delta)$$



$$\left(\frac{-625}{2501}\right)\cos t + \left(\frac{25}{5002}\right)\sin t$$

$$= (R\cos\delta)\cos t + (R\sin\delta)\sin t$$

$= (R\cos\delta)\cos t + (R\sin\delta)\sin t$

$= R\cos(t - \delta)$

$R\cos\delta = \frac{-625}{2501}$ $\qquad$ $R\sin\delta = \frac{25}{5002}$

$\text{amplitude} = R = \sqrt{R^2\cos^2\delta + R^2\sin^2\delta}$

$= \sqrt{\left(\frac{-625}{2501}\right)^2 + \left(\frac{25}{5002}\right)^2}$

(we will do this in 3.7)

2.3 # 11 :

(*) $\quad y' = \frac{(1+\sin t)}{5} y - k$

a) $k = 1/5$ $\qquad$ c) choose various values for K

~~Solve Eqn ⊗ for arbitrary K~~ constant

use slope field instead

(*) is not separable, but it is linear: $1y' + p(t)y = g(t)$

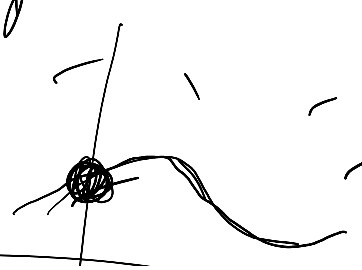
$$1y' - \left(\frac{1+\sin(t)}{5}\right)y = -k$$

Note you are not asked to solve problem.

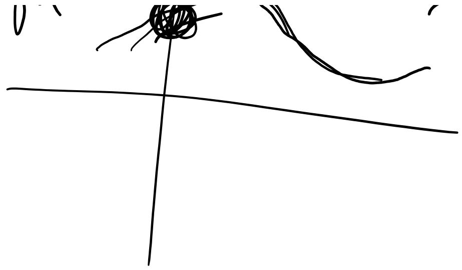
You could, using integrating factor $u(t) = e^{-\frac{1}{5}\int(1+\sin t)\,dt}$

$\boxed{\text{☺ OR use slope field software}}$

recommend method



not slope

method  no slope field

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2.1 # 11 : linear DE [$1y' + p(t)y = g(t)$]

$$1y' + \left(\frac{2}{t}\right)y = \frac{\cos t}{t^2}$$

integrating factor

$$u(t) = e^{\int p(t)\,dt} = e^{\int \frac{2}{t}\,dt}$$

$$= e^{2\ln|t|} = \cancel{e^{\ln}}|t|^2$$

$$= |t|^2$$

Let $u(t) = t^2$ [don't need absolute value even when power is not even]

$$t^2\left[1y' + \frac{2}{t}y\right] = \left[\frac{\cos t}{\cancel{t^2}}\right]\cancel{t^2}$$

$$t^2 y' + 2t\, y = \cos t$$

product rule

$$\boxed{fg' + f'g}$$

$$\int (t^2 y)' \, dt = \int \cos t \, dt$$

$u(t) = t^2$

$y' = \frac{dy}{dt}$

$t' = \frac{dt}{dt} = 1$

FTC

$$\int F'(t)\,dt = F(t) + C$$

$$(F(t) + C)' = F'(t)$$