Any questions?

Monday, February 15, 2021 1:22 PM

2.4: 18b, 10, 17

2.4: 18 b

nonlinear → $y' = \dfrac{-t + \sqrt{t^2+4y}}{2}$, $y(2) = -1$

by part a, this IVP has at least 2 sol'ns

⇒ hypothesis of Thm 2.4.2 does NOT hold

at least one?

① $f(t,y) = \dfrac{-t + \sqrt{t^2+4y}}{2}$ to be cont on a rectangle containing $(2,-1)$

② $\dfrac{\partial f}{\partial y}$ cont on rect conta

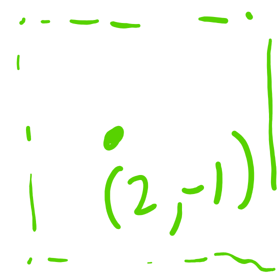
(2) $\frac{\partial f}{\partial y}$ cont on rect contn (2, −1)

$$y' = f(2,-1) = \frac{-2+\sqrt{4-4}}{2} = \frac{-2+\sqrt{0}}{2}$$

(1) Need $\sqrt{t^2+4y}$ exist on open rectangle

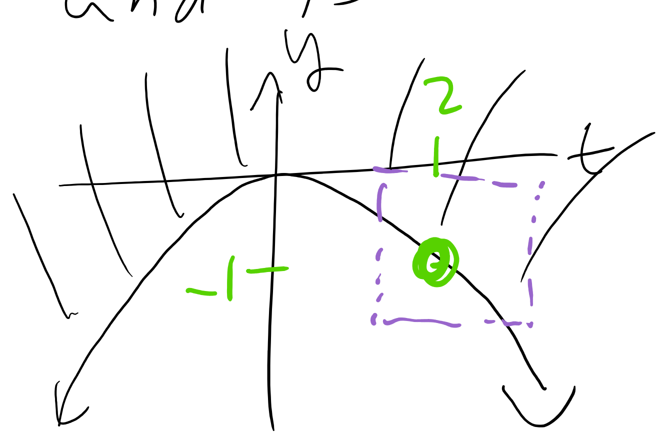
$$t^2+4y > 0$$

$$\frac{4y}{4} > \frac{-t^2}{4}$$



when f exists and is cont on a open rectangle

(2, −1)



there does not exist a rectangle containing (2, -1) s.t f is continuous

Thus hypothesis of Thm 2.4.2 does not hold

since first cond regarding f is not satisfied

[(thus don't need to check 2nd condition $\frac{\partial f}{\partial y}$]

2.4 #10 : Solve

$\frac{dt}{y^2} \frac{dy}{dt} = \frac{2ty^2 \, dt}{y^2}$, $y(0) = y_0$

$$\int y^{-2}\,dy = \int 2t\,dt$$

$$-(-1\, y^{-1}) = -t^2 + C$$

$$y = \frac{1}{-t^2 + C}$$

IVP: $y(0) = y_0 \Rightarrow$

$$y_0 = \frac{1}{0 + C} \Rightarrow C = \frac{1}{y_0}$$

IVP soln: $y = \dfrac{1}{-t^2 + (1/y_0)}$

$y_0 \nearrow$ $\quad t_0$

where does soln exist?

what interval = domain?

what interval = domain?

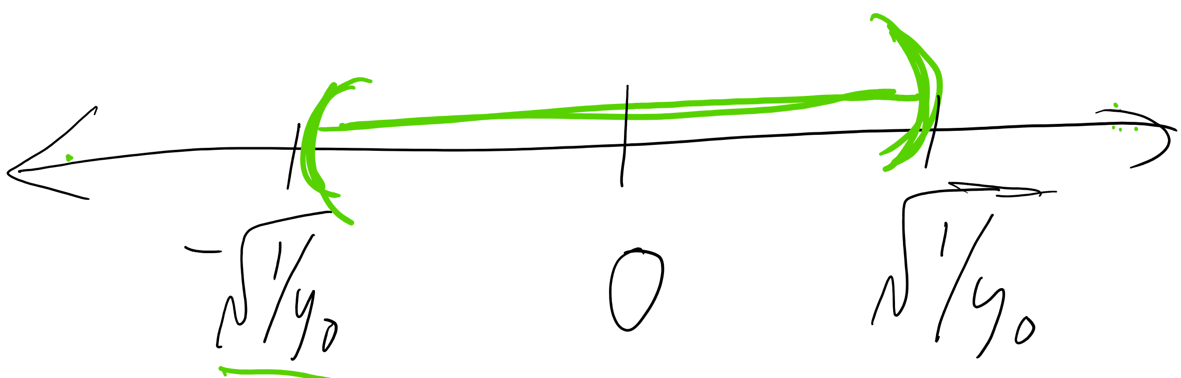
$$-t^2 + \frac{1}{y_0} \neq 0$$

Solve for t since

$$t^2 \neq \frac{1}{y_0} \Rightarrow t = \pm\sqrt{\frac{1}{y_0}}$$

Domain = valid t-values

But choose 1 open interval



$y(0) =$ [illegible]

Domain $\left(-\sqrt{\frac{1}{y_0}}, \sqrt{\frac{1}{y_0}}\right)$

2.4 #17 $y' = y^{1/3}$

2.4 #17 $y' = y^{1/3}$, $y(0) = 0$

a) Find soln if it exists that satisfies ① $y' = y^{1/3}$

② $y(0) = 0$ ③ $y(1) = 1$

Does it exist?

Per Thm 2.4.2

$y' = f(t, y) = y^{1/3}$ is continuos for all t and y

$$Z = \frac{\partial f(t,y)}{\partial y} = \frac{\partial y^{1/3}}{\partial y} = \frac{1}{3} y^{-2/3}$$

$$= \frac{1}{3\sqrt[3]{y^2}}$$

$\Rightarrow \sqrt[3]{y^2}$

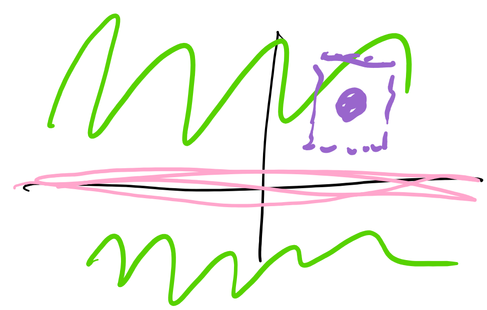
$\frac{\partial f}{\partial y}$ is cont if $y \neq 0$

$\Rightarrow$ IVP $y' = y^{1/3}, y(0) = 0$ no info

$\Rightarrow$ IVP $y' = y^{1/3}$ $y(1) = 1$ has a unique soln

$1 \neq 0$

so f & $\frac{\partial f}{\partial y}$ are cont on rectangle about $(1,1)$



Since soln is unique don't need to worry about algebra loosing solns

$$\frac{dt}{y^{1/3}} \frac{dy}{dt} = \frac{y^{1/3}\, dt}{y^{1/3}} \Rightarrow \int y^{-1/3}\, dy = \int dt$$

$y^{1/3}$ at ... $y^{1/3}$ ...

$$\frac{2}{3}\left(\frac{3}{2} y^{2/3}\right) = \frac{2}{3} t + C$$

$$\boxed{y = \pm\left(\frac{2}{3} t + C\right)^{3/2}}$$

IVP: $y(1) = 1$: $1 = +\left(\frac{2}{3} + C\right)^{3/2}$

$$1 = \left(\frac{2}{3} + C\right) \Rightarrow C = \frac{1}{3}$$

Unique soln to IVP $y(1) = 1$ is

$$\boxed{y = +\left(\frac{2}{3} t + \frac{1}{3}\right)^{3/2}}$$

Does this pass thru $(0,0)$?

NO since $0 \neq \left(\frac{2}{3}(0)+\frac{1}{3}\right)^{3/2}$

a) there does not exist soln that passes thru both (1,1) and (0,0)

Note didn't solve IVP $y' = y^{1/3}$, $y(0) = 0$ since has infinite # of solns

$$y = \begin{cases} 0 & \text{if } 0 \le t < t_0 \\ \pm\left[\frac{2}{3}(t - t_0)\right]^{3/2} & \text{if } t \ge t_0 \end{cases}$$

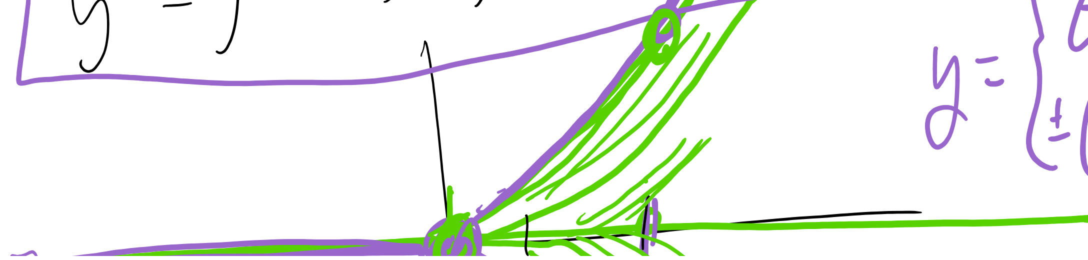
but then need to check all of these ∞ # of solns to see if any pass thru (1,1) we can do this, but boring algebra

$$1 = \pm\left(\frac{2}{3}(1-t_0)\right)^{3/2} \quad \text{if } 1 > t_0$$

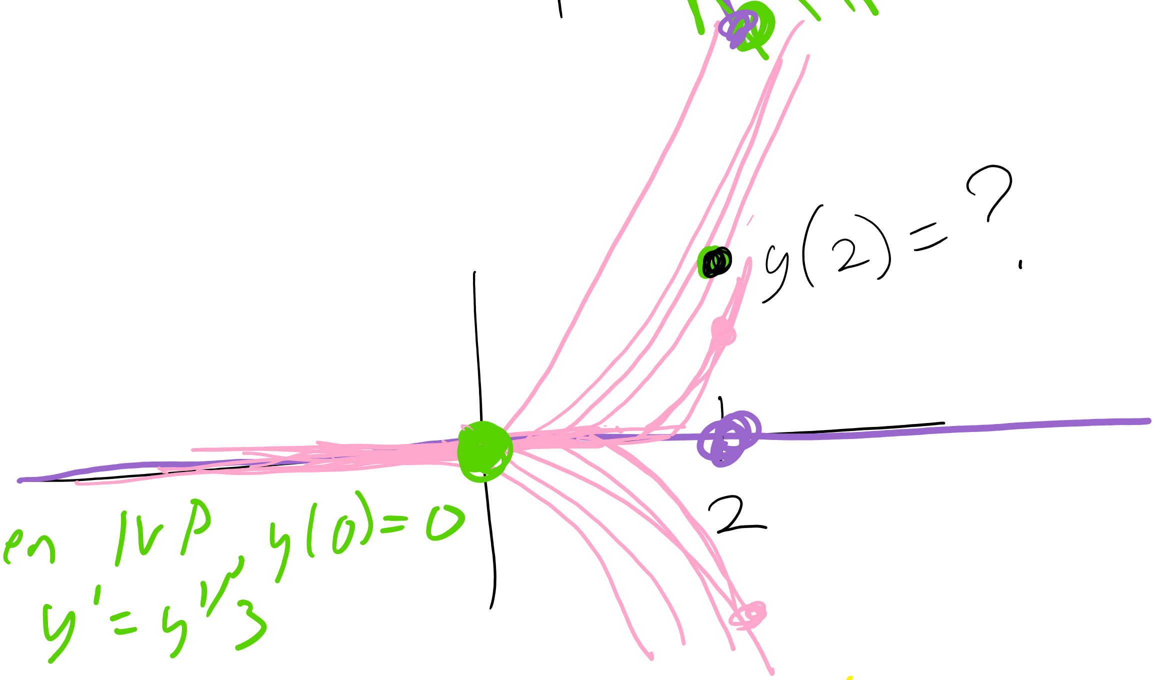
can solve for t_0 and see if $1 > t_0$

No soln for $t_0 > 1$

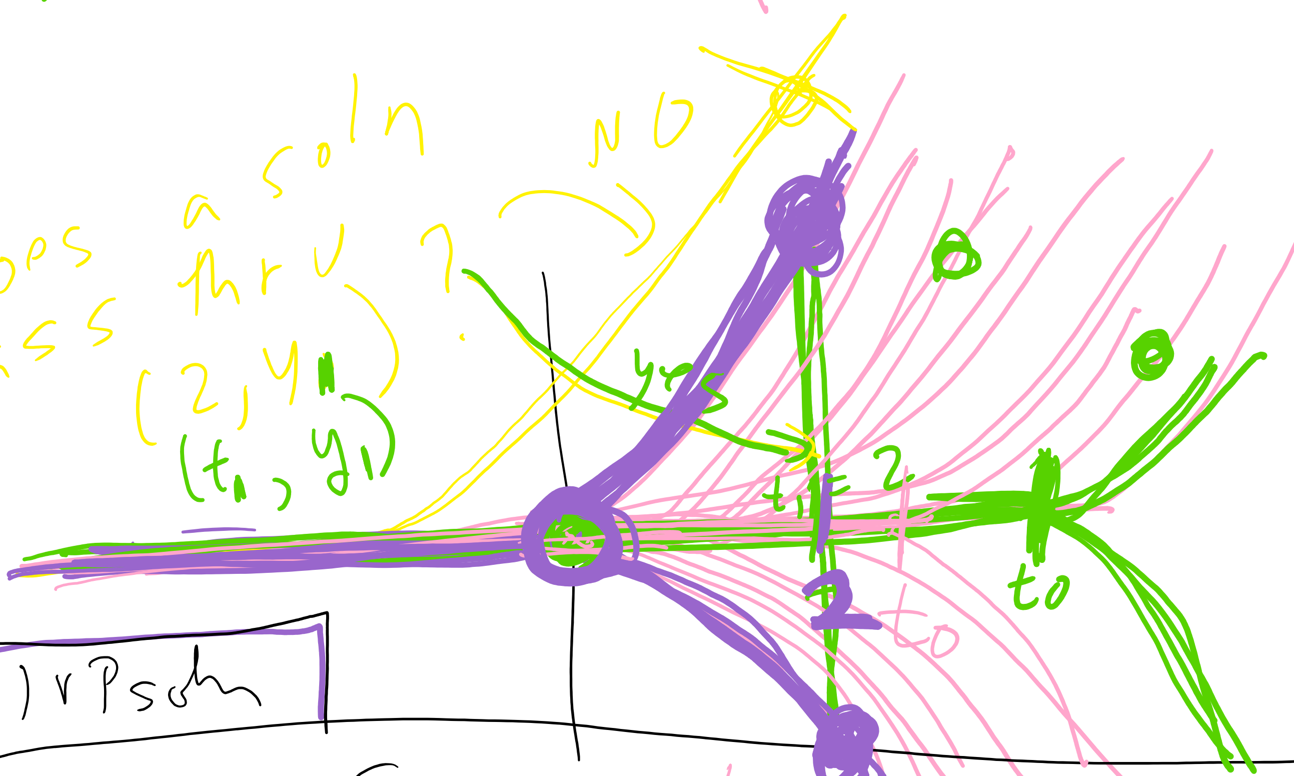
17c $y' = y^{1/3}$, $y(0) = 0$



$y = \begin{cases} 0 \\ \pm(\quad)^{3/2}, \; t > \end{cases}$



Given IVP, $y(0)=0$
$y' = y^{1/3}$



Does a soln pass thru $(2, y_1)$?
(t_1, y_1)

IVP soln

$$y = \begin{cases} 0 & t < t_0 \\ \pm\left(\frac{2}{3}(t - t_0)\right)^{3/2} & t > t_0 \end{cases}$$

$$y = \pm\left(\frac{2}{3}(t - t_0)\right)^{3/2} \quad t \geq t_0$$