

Any questions?

LS Quiz 2 due date moved to tomorrow

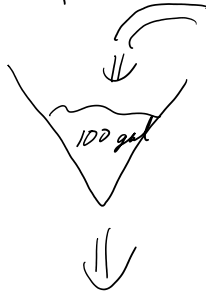
(will avoid Monday due dates)

Quizzes will be due Tuesdays (with grace period of one day)

Questions?

2.3 #3 water tank problem

initially 100 gal of water w/ 50 oz of salt

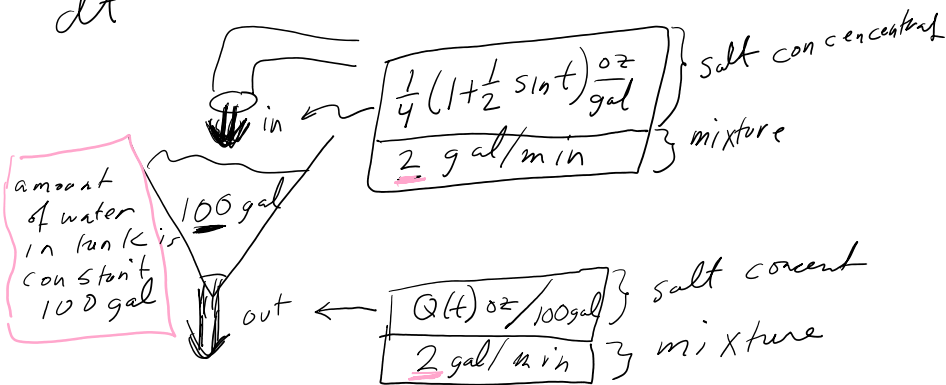


IVP: DE + initial value

Let $Q(t)$ = amount of salt in oz in tank at time t in minutes

initial value: $Q(0) = 50$ oz

$$\frac{dQ}{dt} = \text{rate in} - \text{rate out}$$



$$\frac{dQ}{dt} = \left(\frac{1}{4} \left(1 + \frac{1}{2} \sin t \right) \frac{\text{oz}}{\text{gal}} \right) (2 \frac{\text{gal}}{\text{min}}) - \left(\frac{Q}{100} \frac{\text{oz}}{\text{gal}} \right) (2 \frac{\text{gal}}{\text{min}})$$

$$\frac{dQ}{dt} = \text{rate in} - \text{rate out}$$

$$= \left(\frac{1}{4} (1 + \frac{1}{2} \sin t) \frac{0.2}{\text{gal}} \right) (2 \frac{\text{gal}}{\text{min}}) - \left(\frac{Q}{100 \text{ gal}} \right) (2 \frac{\text{gal}}{\text{min}})$$

\uparrow $\frac{0.2}{\text{min}}$
 salt concentration at time t

$$\frac{dQ}{dt} = \left(\frac{1}{4} (2 + \sin t) - \frac{Q}{50} \right) \quad Q(0) = 50$$

started w 50oz of salt in tank

not separable

but it is linear

$$1Q' + \frac{Q}{50} = \frac{1}{4} (2 + \sin t)$$

integrating factor $\int p(t) dt$

$$u(t) = e^{\int \frac{1}{50} dt} = e^{t/50}$$

since $p(t) = \frac{1}{50}$

$$e^{t/50} [Q' + \frac{Q}{50}] = \left[\frac{1}{2} (2 + \sin t) \right] e^{t/50}$$

$$e^{t/50} Q' + \frac{1}{50} e^{t/50} Q = \frac{1}{2} (2e^{t/50} + \sin t e^{t/50})$$

check product rule ✓

$$\int (e^{t/50} \cdot Q)' dt = \frac{1}{2} \int (2e^{t/50} + \sin t e^{t/50}) dt$$

$$e^{t/50} Q = \frac{1}{2} \int (2e^{t/50} + \sin t e^{t/50}) dt$$

$$= \frac{1}{2} \left[100e^{t/50} + \int \sin t e^{t/50} dt \right]$$

$$= \frac{1}{2} \left[100e + \int \sin t \, dt \right]$$

$$\int \sin t e^{t/50} dt = (\sin t)(50e^{t/50})$$

| | |
|------------------|-------------------------|
| $u = \sin t$ | $dv = e^{t/50}$ |
| $du = \cos t$ | $v = 50e^{t/50}$ |
| $du^2 = -\sin t$ | $\int v = 2500e^{t/50}$ |

$$- (\cos t)(2500e^{t/50})$$

$$+ \int (-\sin t)(2500e^{t/50}) dt$$

$$\int (\sin t)(e^{t/50}) dt = 50e^{t/50} \sin t - 2500e^{t/50} \cos t - 2500 \int (\sin t)(e^{t/50}) dt$$

$$2501 \int (\sin t) e^{t/50} dt = 50e^{t/50} \sin t - 2500e^{t/50} \cos t$$

$$\int \sin t e^{t/50} dt = 50e^{t/50} \sin t - 2500e^{t/50} \cos t$$

plug in, etc.