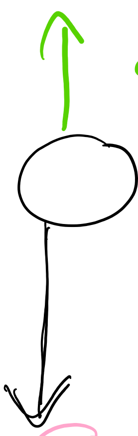
Monday, January 25, 2021 1:19 PM

§1.1: Falling ball example



Force due to air resistance

= proportional to velocity

$= \gamma v$

γ = proportionality constant

v = velocity

gravitational force

mg

m = mass

$g = 9.8\ m/sec^2$

$32\ ft/sec^2$

(+) ← positive direction pts down

Total force $= F_{gravity} + F_{air\ resistance}$

$= +mg - \gamma v$

$+mg$: acts in positive direction

$-\gamma v$: acts in direction opposite of ball motion

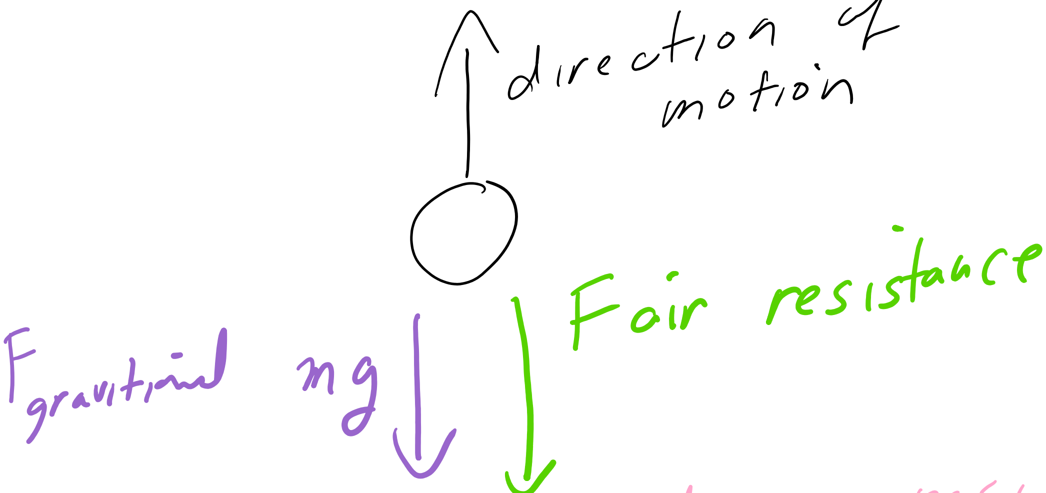
ball falling ⟹ air resistance acts in upward negative direction

Newton's 2nd law

$$F = ma = m\frac{dv}{dt}$$

$$\boxed{m\frac{dv}{dt} = mg - \gamma v}$$

Ball is thrown up



(+) ← down = positive direction

$$m\frac{dv}{dt} = +mg - \gamma v$$

(since $v < 0 \Rightarrow -\gamma v > 0$)

air resistance force is positive since acting in positive direction downward

$$v = \frac{dx}{dt} \approx \frac{x_2 - x_1}{t_2 - t_1} < 0$$

$x(t)$ = location at time t

$$v < 0 \Rightarrow \gamma v < 0$$

so we need $-\gamma v > 0$

Ball moving up or down is modeled by DE

$$m\frac{dv}{dt} = mg - \gamma v$$

EX: Suppose $m = 10\,kv$, $\gamma = 2\ m/sec^2$

$$\frac{10}{10}\frac{dv}{dt} = \frac{10}{10}(9.8) - \frac{2}{10}v$$

$$\frac{dv}{dt} = 9.8 - \frac{1}{5}v$$

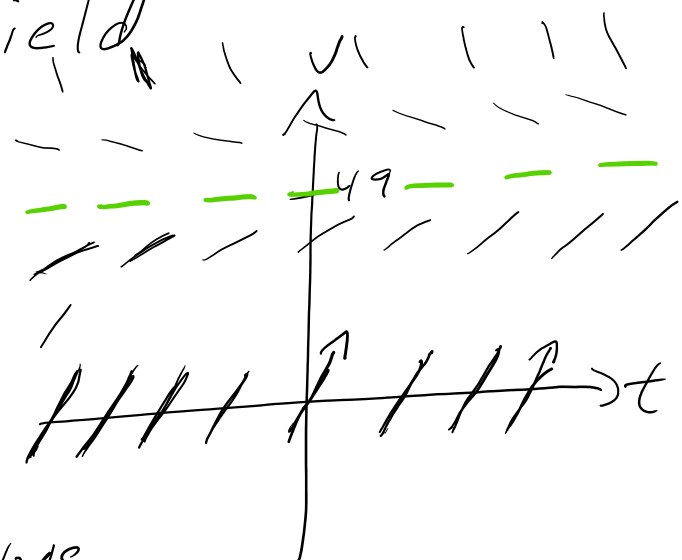
To determine how velocity (v) depends on time (t)

(1) Solve DE: See Wednesday's class

② Draw direction field

= draw small portions of tangent lines in the t-v plane

= slope field



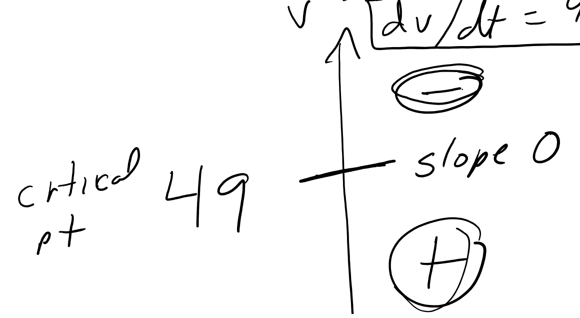
t	v	$\frac{dv}{dt} = 9.8 - \frac{v}{5}$
	0	9.8
	49	0
	40	small positive slope
	50	small negative slope
	60	steeper negative slope

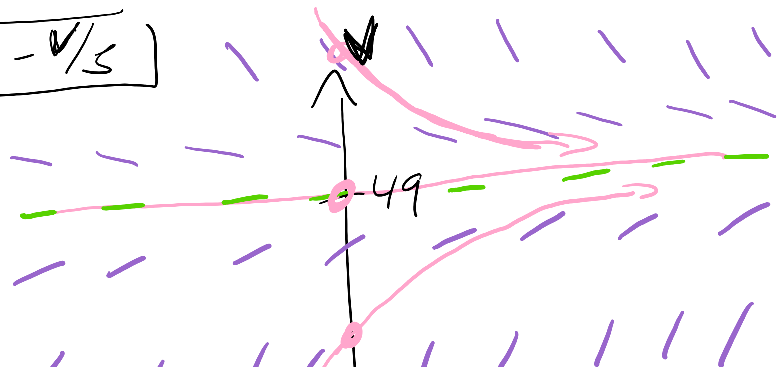
$\frac{dv}{dt}$ does not depend on t in this example

$$\frac{dv}{dt} = 0 = 9.8 - \frac{v}{5}$$

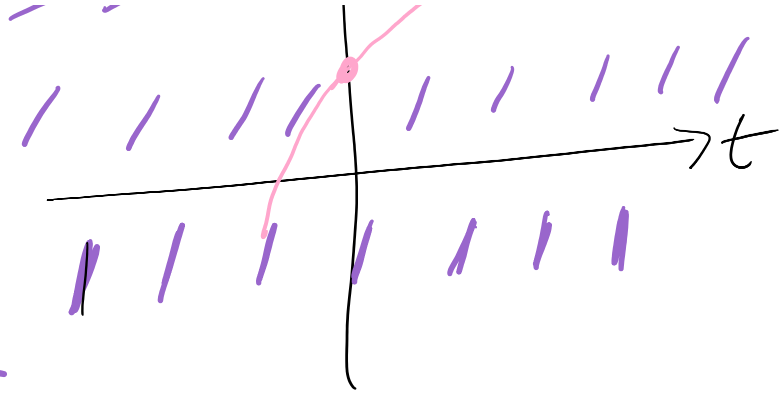
$$v = (9.8)5 = 49$$

⇒





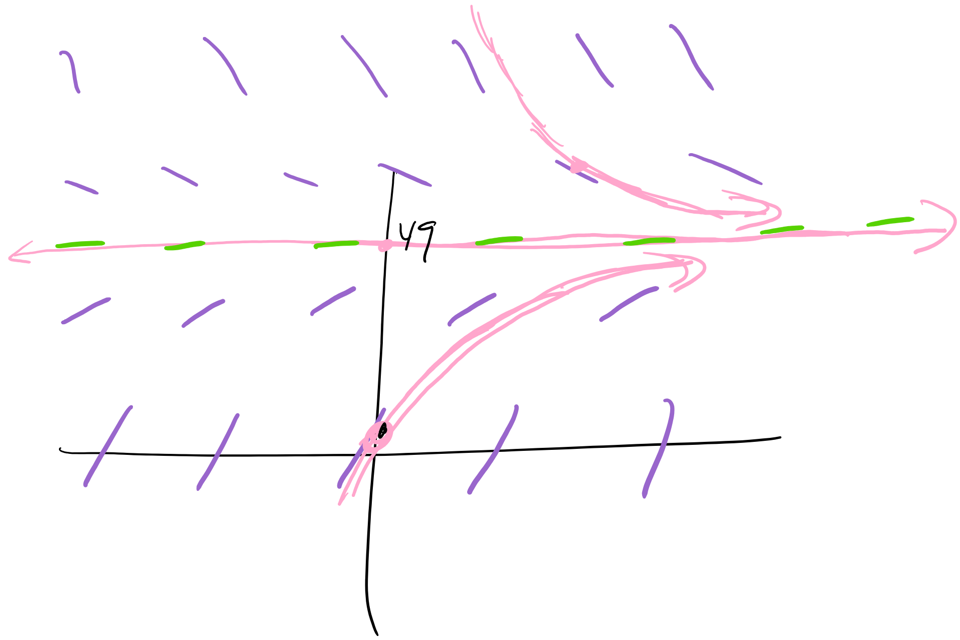
pt



compare to calc 1

as $t \to \infty$

$v \to 49$ m/sec



* **Equilibrium solution**

= constant sol'n

($v = k$) ie $v = 49$

$\Updownarrow$

$$\boxed{\frac{dv}{dt} = 0}$$

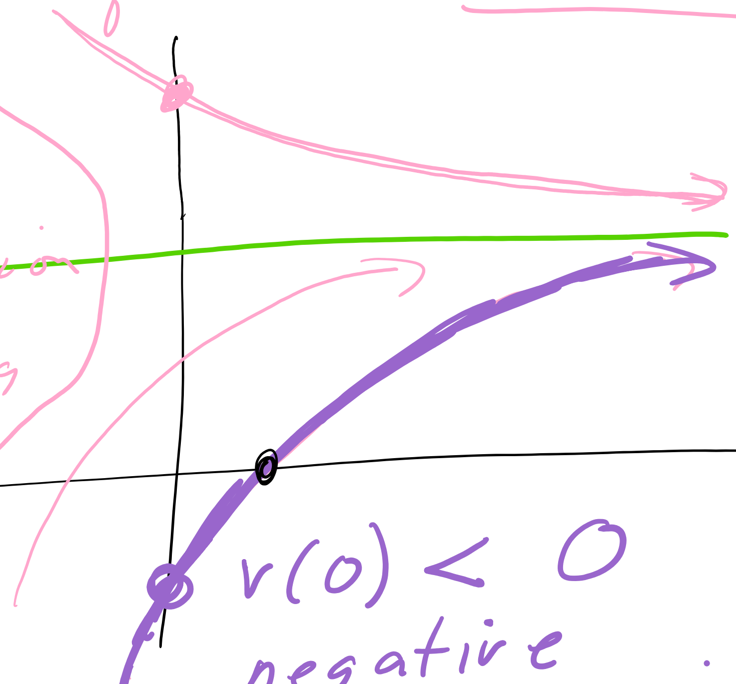
ie $\frac{dv}{dt} = 9.8 - \frac{v}{5} = 0$

(solve $\frac{dv}{dt} = 9.8 - \frac{v}{5} = 0$ to find equil soln $\boxed{v = 49}$

$v(0) > 49$

velocity > 0 so ball is moving in positive direction and from graph velocity is getting smaller and $\to 49$

ball thrown down



$v(0) < 0$

negative ball moves in negative up direction at $t = 0$

(ie ball is thrown up