1.3 #19:

Show $u_1(x,y) = \cos x \cosh y$

$u_2(x,y) = \ln(x^2+y^2)$

are sol'ns to PDE: $u_{xx} + u_{yy} = 0$

Recall $u_{xx} = \underbrace{\frac{\partial}{\partial x}\frac{\partial}{\partial x}}_{\frac{\partial^2}{\partial x^2}}(u)$ ← Treat y as a constant

For u_2

$$\frac{\partial^2}{\partial x^2}\left(\ln(x^2+y^2)\right) = \frac{\partial}{\partial x}\left(\frac{1}{x^2+y^2} \quad \partial(x^2+y^2)\right)$$

$$\frac{\partial}{\partial x}\frac{\partial}{\partial x}\left(\ln\left(x^2+y^2\right)\right) = \frac{\partial}{\partial x}\left(\frac{1}{\left(x^2+y^2\right)}\cdot 2x\right)$$

use quotient rule $\frac{2x}{x^2+y^2}$

or product rule $(x^2+y^2)^{-1}\cdot 2x$

$$= \left[-\left(x^2+y^2\right)^{-2}(2x)\right](2x) + \left(x^2+y^2\right)^{-1}(2)\Big]$$

$$= \frac{(-2x)(2x) + 2\left(x^2+y^2\right)}{\left(x^2+y^2\right)^2}$$

$\frac{\partial}{\partial x} = \frac{d}{dx}$ where y is a constant

$$= \frac{2\left(x^2+y^2\right) - 4x^2}{\left(x^2+y^2\right)^2} = \boxed{\frac{2y^2-2x^2}{\left(x^2+y^2\right)^2}}$$

$$u_{xx} = \frac{2y^2-2x^2}{(x^2+y^2)^2}$$

$$u_{yy} = \frac{\partial}{\partial y}\frac{\partial}{\partial y}\left[\ln(x^2+y^2)\right] = \frac{2x^2-2y^2}{(x^2+y^2)^2}$$

Verify that $u_{xx} + u_{yy} = 0$ when $u = \ln(x^2+y^2)$

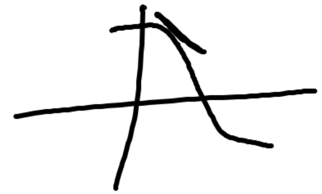
To show something is a soln, plug in (into LHS)

$$\underbrace{\frac{2y^2-2x^2}{(x^2+y^2)^2}} + \underbrace{\frac{2x^2-2y^2}{(x^2+y^2)^2}} = \frac{\cancel{2y^2}-\cancel{2x^2}+\cancel{2x^2}-\cancel{2y^2}}{(x^2+y^2)^2} = 0 \checkmark$$

$\Rightarrow u_2 = \ln(x^2+y^2)$ is a soln to $u_{xx}+u_{yy}=0$

For u_1, also plug in to check answer

$\to u_{xx} + u_{yy} = 0$



Find u_{xx} & u_{yy}

$$u_1(x,y) = ((\cos x)(\cosh y))$$

$$\frac{\partial}{\partial x}\frac{\partial}{\partial x}(u_1(x,y)) = \underbrace{\cosh y}_{\text{constant}} \cdot \left(\frac{\partial^2}{\partial x^2}\cos x\right) = etc$$

$$\frac{\partial^2}{\partial y^2}((\cos x)(\cosh y)) = \cos x\left[\frac{\partial^2}{\partial y^2}(\cosh y)\right]$$

∴ look up formula for cosh y https://socratic.org/qu
what-is-the-derivative
-x

Generic question regarding constants:

$$\left.\begin{array}{l} f(x) + C \\ f(x) - C \end{array}\right\}$$ same thing (if sloppy)

C can swallow negative sign

recall can use C to mean more than 1 thing

Both are acceptable answers

1.2 1a $\frac{dy}{dt} = -y + 5$ $\quad y(0) = y_0$

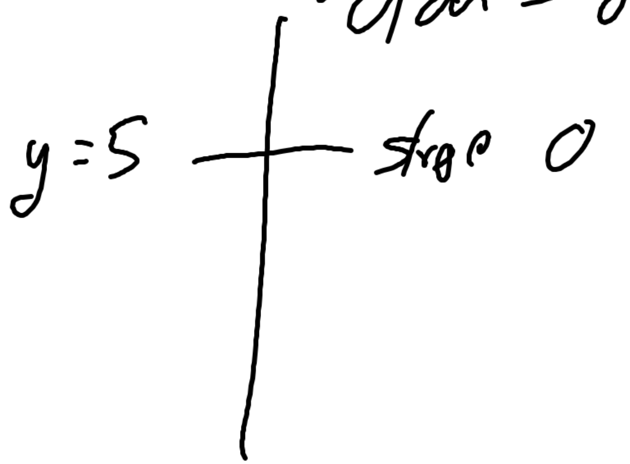
① Find equilibrium soln if any

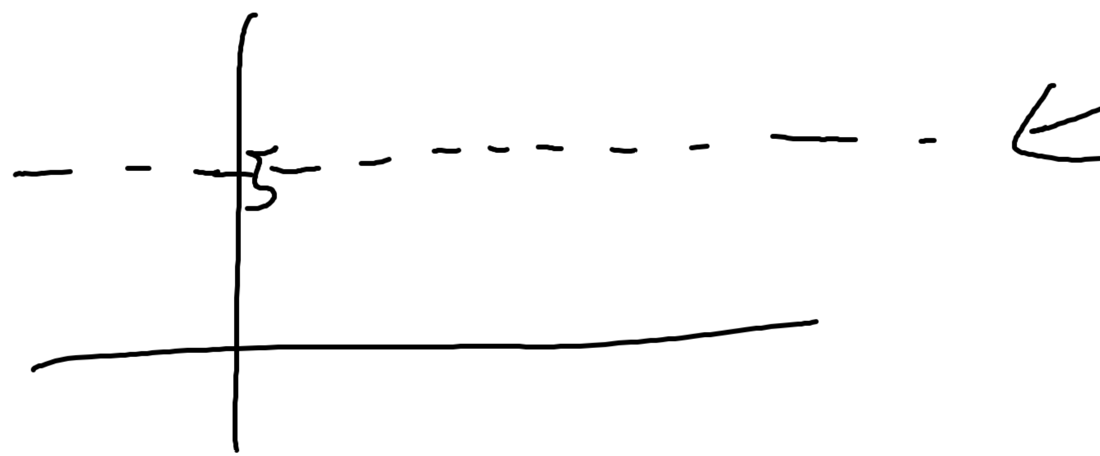
$y = k \Longleftrightarrow y' = 0$

$\Updownarrow$ slopes are zero

$0 = -y + 5 \Rightarrow y = 5$

$dy/dt = -y + 5$





Solve $\frac{dy}{dt} = -y + 5$ using separate variables

$$\int \frac{dy}{-y+5} = \int dt$$

$$+\ln|-y+5| = -t + C$$

etc.