Math 3600 Differential Equations Exam #1
March 2, 2016 SHOW ALL WORK

[20] 1.) Solve $y'' - 6y' + 9 = 0$, $y(0) = 2$, $y'(0) = 4$.

Answer: ____

2.) Circle T for true and F for false.

[4] 2a.) The equation $ln(t)y' = \frac{t}{t+1} - y(sint^2)$ is a linear differential equation.

T F

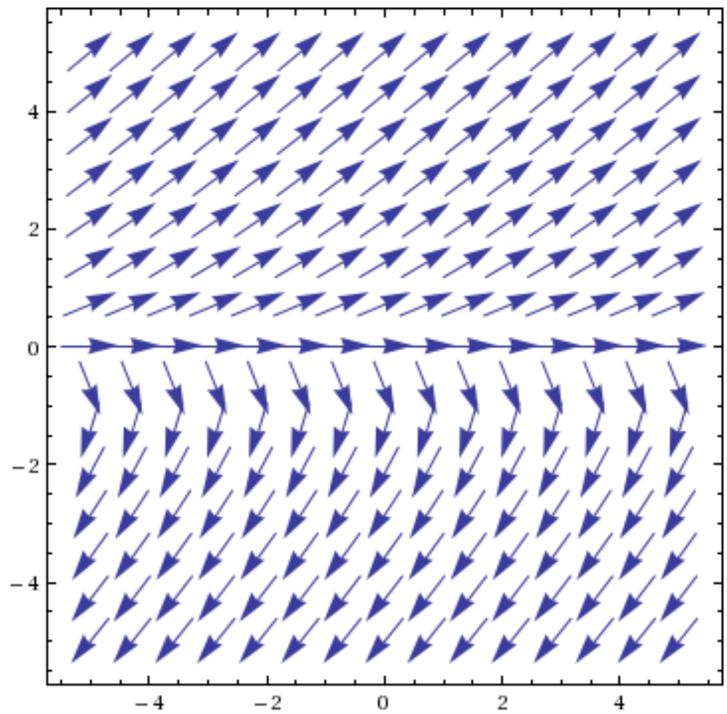
[4] 2b.) The equation $y' + y = y^2$ is a linear differential equation. T F

[4] 2c.) Suppose $y = \phi_1(t)$ and $y = \phi_2(t)$ are solutions to $ay'' + by' + cy = 0$. If $y = h(t)$ is also a solution to $ay'' + by' + cy = 0$, then there exists constants c_1 and c_2 such that $h(t) = c_1\phi_1(t) + c_2\phi_2(t)$. T F

[4] 2d.) Suppose $y = \phi_1(t)$ and $y = \phi_2(t)$ are linearly independent solutions to $ay'' + by' + cy = 0$. If $y = h(t)$ is also a solution to $ay'' + by' + cy = 0$, then there exists constants c_1 and c_2 such that $h(t) = c_1\phi_1(t) + c_2\phi_2(t)$. T F

[4] 3.) By giving a specific counter-example, prove that $y = ln(x)$ is not a linear function.

[20] 4.)



4a.) Circle the differential equation whose direction field is given above.

i.) $y' = y^2$ ii.) $y' = (y+2)^2$ iii.) $y' = y(y+2)^2$

iv.) $y' = y(y+2)$ v.) $y' = \frac{y}{y+2}$ vi.) $y' = \frac{y+2}{y}$

vii.) $y' = \frac{y^2}{t}$ viii.) $y' = t\sqrt{y}$ ix.) $y' = ty^2$

4b.) Draw the solution to the differential equation whose direction field is given above that satisfies the initial condition $y(1) = -3$.

4c.) Does the differential equation whose direction field is given above have any equilibrium solutions? If so, state whether they are stable, semi-stable or unstable.

[20] 5.) Solve: $y' = e^{4t} - \frac{y}{t}$

Answer: ______________________________

[20] 6.) Choose one of the following 2 problems. If you do not choose your best problem, I will substitute the other problem, but with a 2 point penalty (if it improves your grade). Circle the letter corresponding to your chosen problem: A B

6A.) Show that $\phi(t) = \sum_{k=2}^{\infty} \frac{3(-1)^k\ t^k}{k!}$ converges for all t and show that $\phi(t) = \sum_{k=2}^{\infty} \frac{3(-1)^k\ t^k}{k!}$ is a solution to $y' = 3t - y$.

6B.) Show by induction that for Picard's iteration method, $\phi_n(t) = \sum_{k=1}^{n} \frac{3(-1)^{k+1}\ t^{k+1}}{(k+1)!}$ approximates the solution to the initial value problem, $y' = 3t - y$, $y(0) = 0$ where $\phi_1(t) = \frac{3t^2}{2}$. You may use the proof outline below or write it from scratch.

Proof by induction on n.

For $n = 1$, $\sum_{k=1}^{1} \frac{3(-1)^{k+1}\ t^{k+1}}{(k+1)!} =$

Suppose for $n = j$, $\phi_{j-1}(t) = \sum_{k=1}^{j-1} \frac{3(-1)^{k+1}\ t^{k+1}}{(k+1)!}$

Then by Picard's iteration method, $\phi_j =$