Solve $y'' - 4y' - 5y = 4sin(3t)$, $y(0) = 6$, $y'(0) = 7$.

1.) Find the general solution to $y'' - 4y' - 5y = 0$:

Guess $y = e^{rt}$ for HOMOGENEOUS equation:

$y' = re^{rt}$, $y' = r^2e^{rt}$

$y'' - 4y' - 5y = 0$

$r^2e^{rt} - 4re^{rt} - 5e^{rt} = 0$

$e^{rt}(r^2 - 4r - 5) = 0$

$e^{rt} \neq 0$, thus can divide both sides by e^{rt}:

$$r^2 - 4r - 5 = 0$$

$(r+1)(r-5) = 0$. Thus $r = -1, 5$.

Thus $y = e^{-t}$ and $y = e^{5t}$ are both solutions to HOMOGENEOUS equation.

Thus the general solution to the 2nd order linear HOMOGENEOUS equation is

$$y = c_1e^{-t} + c_2e^{5t}$$

2.) Find a solution to $ay'' + by' + cy = 4sin(3t)$:

Guess $y = Asin(3t) + Bcos(3t)$

$y' = 3Acos(3t) - 3Bsin(3t)$

$y'' = -9Asin(3t) - 9Bcos(3t)$

$y'' - 4y' - 5y = 4sin(3t)$

y''	$-9Asin(3t)$	$-$	$9Bcos(3t)$	helps to be
$-4y'$	$12Bsin(3t)$	$-$	$12Acos(3t)$	organized
$-5y$	$-5Asin(3t)$	$-$	$5Bcos(3t)$	

$$(12B - 14A)sin(3t) + (-14B - 12A)cos(3t) = 4sin(3t) + 0\,cos(3t)$$

Since $\{sin(3t), cos(3t)\}$ is a linearly independent set:

$12B - 14A = 4$ and $-14B - 12A = 0$

Thus $A = -\frac{14}{12}B = -\frac{7}{6}B$ and

$12B - 14(-\frac{7}{6}B) = 12B + 7(\frac{7}{3}B) = \frac{36+49}{3}B = \frac{85}{3}B = 4$

Thus $B = 4(\frac{3}{85}) = \frac{12}{85}$ and $A = -\frac{7}{6}B = -\frac{7}{6}(\frac{12}{85}) = -\frac{14}{85}$

Thus $y = (-\frac{14}{85})sin(3t) + \frac{12}{85}cos(3t)$ is one solution to the non-homogeneous equation.

Thus the general solution to the 2nd order linear nonhomogeneous equation is

$$y = \underbrace{c_1e^{-t} + c_2e^{5t}}_{\text{gen homog soln}} - \underbrace{(\tfrac{14}{85})sin(3t) + \tfrac{12}{85}cos(3t)}_{\text{a single non homog}}$$

5.1 Review of Power Series.

Definition: $\Sigma_{n=0}^{\infty} a_n(x - x_0)^n = lim_{n\to\infty}\Sigma_{n=0}^{k} a_n(x - x_0)^n$

$\boxed{\text{Taylor's Theorem}}$

Suppose f has $n+1$ continuous derivatives on an open interval containing a. Then for each x in the interval,

error term

$$f(x) = \underbrace{\left[\sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k\right]}_{\text{polynomial approx}} + R_{n+1}(x)$$

where the error term $R_{n+1}(x)$ satisfies $R_{n+1}(x) = \dfrac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$ for some c between a and x.

The *infinite* Taylor series converges to f,

$$\boxed{f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k} \text{ if and only if } \lim_{n\to\infty} R_n(x) = 0.$$

Defn: The function f is said to be **analytic** at a is its Taylor series expansion about $x = a$ has a positive radius of convergence.

1.) $\Sigma_{n=0}^{\infty} a_n(x - x_0)^n$ converges at the point x if and only if $lim_{n\to\infty}\Sigma_{n=0}^{k} a_n(x - x_0)^n$ ← finite # exists at the point x.

2.) $\Sigma_{n=0}^{\infty} a_n(x - x_0)^n$ converges absolutely at the point x if and only if $\Sigma_{n=0}^{\infty}|a_n||x - x_0|^n$ converges at the point x

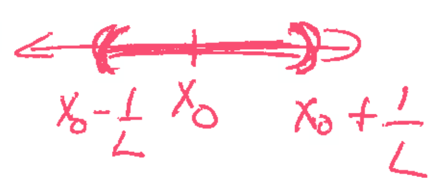
If a series converges absolutely, then it also converges.

3.) Ratio test for absolute convergence:

Let $L = lim_{n\to\infty}|\frac{a_{n+1}}{a_n}|$

$lim_{n\to\infty}|\frac{a_{n+1}\cancel{(x-x_0)^{n+1}}}{a_n\cancel{(x-x_0)^n}}| = |x - x_0| lim_{n\to\infty}|\frac{a_{n+1}}{a_n}| = |x - x_0|L < 1 \Rightarrow$ absolute converges

The power series converges at at the value x if $|x - x_0| < \frac{1}{L}$



The power series diverges at the value x if $|x - x_0| > \frac{1}{L}$

The ratio test give no info at the value x if $|x - x_0| = \frac{1}{L}$

Note $\frac{1}{L}$ is the **radius of convergence**.