nonhomog if $g_i \neq 0$

To solve linear DE $ay'' + by' + cy = g_1 + g_2 + g_3$

homog

Step 1: Solve homogeneous version: $ay'' + by' + cy = 0$ implies
$ar^2 + br + c = 0$ implies ... $\boxed{y = c_1\phi_1 + c_2\phi_2}$.

Step 2a: Find one non-homogeneous solution, $y = f_1$, to $ay'' + by' + cy = g_1$
Step 2b: Find one non-homogeneous solution, $y = f_2$, to $ay'' + by' + cy = g_2$
Step 2c: Find one non-homogeneous solution, $y = f_3$, to $ay'' + by' + cy = g_3$

Step 3: Combine all solutions to create the general solution to the non-homogeneous DE:

$$\boxed{y = c_1\phi_1 + c_2\phi_2 + f_1 + f_2 + f_3}$$

Last step: If IVP, plug in initial values to find the constants c_1 and c_2.

Guess a possible non-homog soln for the following DEs:

$y = e^{rt}$

Note homogeneous solution to $y'' + 2y' + y = 0$ is
$y = c_1e^{-t} + c_2te^{-t}$ since $r^2 + 2r + 1 = (r+1)(r+1) = 0 \Rightarrow r = -1, -1$

1.) $y'' + 2y' + y = 4e^{2t}$

Guess: $y = Ae^{2t}$ ← plug in to solve for undetermined coeff A

2.) $y'' + 2y' + y = 4e^{t}$

Guess: $y = Ae^{t}$

3.) $y'' + 2y' + y = 4e^{-t}$

Guess: $y = At^2e^{-t}$

$y = Ae^{-t}$ is a homog soln $\Rightarrow$ wrong guess

Multiply by t $\Rightarrow$ $y = Ate^{-t}$ is a homog soln $\Rightarrow$ wrong guess
(plug in $\Rightarrow$ $0 = 4e^{-t}$ $\Rightarrow$ no soln)

Multiply by t $\Rightarrow$ $y = At^2e^{-t}$ is Not homog

No soln $0 + 2A + At = t$ $y = At + B \Rightarrow y' = A$ constant fn $\Rightarrow y'' = 0$

4.) $y'' + 2y' + y = t$

Guess: $y = At + B$

degree 1 polynomial $\Rightarrow$ guess is a degree 1 polynomial

5.) $y'' + 2y' + y = t + 1$

Guess: $y = At + B$

6.) $y'' + 2y' + y = 4sin(2t)$ $y = A\sin(2t) + B\cos(2t)$ $\Rightarrow y' = 2A\cos(2t) - 2B\sin(2t)$

Guess: $y = A\sin(2t) + B\cos(2t)$

7.) $y'' + 2y' + y = 4sin(2t) + 5cos(2t)$

Guess: $y = A\sin(2t) + B\cos(2t)$

8.) $y'' + 2y' + y = 4sin(2t) + 5cos(3t)$

Guess for step 2a: $y = A\sin(2t) + B\cos(2t)$

Guess for step 2b: $y = A_2\sin(3t) + B_2\cos(3t)$

9.) $y'' + 2y' + y = 4sin(2t) + t + 1$

Guess for step 2a: $y = A\sin(2t) + B\cos(2t)$

Guess for step 2b: $y = At + B$

10.) $y'' + 2y' + y = 4t\sin(2t)$ product $\Rightarrow$ guess is a product

Guess: $y = (At + B)(C\sin(2t) + D\cos(2t))$

no y' term $\Rightarrow$ no cosine term

11.) $y'' + y = 4sin(2t)$

Guess: $y = A\sin(2t)$

can add $B\cos 2t$ term, but don't need it since no y' term

But $y = A\sin t + B\cos t$ works $\Rightarrow B = 0$

12.) $y'' + y = 4sin(t)$

Guess: $y = At\sin(t) + Bt\cos t$

$y = t(A\sin t + B\cos t)$

homog $y'' + y = 0$ Non simplified homog soln multiplied by t

$r^2 + 1 = 0 \Rightarrow r = \pm\sqrt{-1} = \pm i \Rightarrow$ homog soln $y = c_1 \cos t + c_2 \sin(t)$

$y = At\sin t$

$\Rightarrow y' = A\sin t + At\cos t$ ✱ product rule ✱

$\Rightarrow y'' = \underline{A\cos t} + \underline{A\cos t} - At\sin t$

To solve linear DE $ay'' + by' + cy = g_1 + g_2 + g_3$ nonhomog if $g_i \neq 0$

homog

Step 1: Solve homogeneous version: $ay'' + by' + cy = 0$ implies
$ar^2 + br + c = 0$ implies ... $y = c_1\phi_1 + c_2\phi_2$.

Step 2a: Find one non-homogeneous solution, $y = f_1$, to $ay'' + by' + cy = g_1$
Step 2b: Find one non-homogeneous solution, $y = f_2$, to $ay'' + by' + cy = g_2$
Step 2c: Find one non-homogeneous solution, $y = f_3$, to $ay'' + by' + cy = g_3$

Step 3: Combine all solutions to create the general solution to the non-homogeneous DE:

~~$y = (At^2 + Bt + C)e^{at}$~~

$$\boxed{y = c_1\phi_1 + c_2\phi_2 + f_1 + f_2 + f_3}$$

Last step: If IVP, plug in initial values to find the constants c_1 and c_2.

Guess a possible non-homog soln for the following DEs:

Note homogeneous solution to $y'' + 2y' + y = 0$ is
$y = c_1e^{-t} + c_2te^{-t}$ since $r^2 + 2r + 1 = (r+1)(r+1) = 0$

1.) $y'' + 2y' + y = 4e^{2t}$

Guess: $y = Ae^{2t}$ then plug in to solve for A

2.) $y'' + 2y' + y = 4e^{t}$

Guess: $y = Ae^{t}$

3.) $y'' + 2y' + y = 4e^{-t}$

Guess: $y = At^2e^{-t}$

since $y = e^{-t}$ is a homog, $y = Ae^{-t}$ is a wrong guess
plug in $\Rightarrow 0$

multiply by t $\quad y = Ate^{-t}$ is also a wrong guess since it is also a homog soln

multiply by t $\quad y = At^2e^{-t}$ ← correct guess

4.) $y'' + 2y' + y = t$ $y = At+B$ $\Rightarrow y' = A \Rightarrow y'' = 0$

Guess: $y = At + B$ degree 1 polynomials

$\Rightarrow$ guess is a degree 1 polynomial

5.) $y'' + 2y' + y = \boxed{t+1}$

Guess: $y = At + B$

6.) $y'' + 2y' + y = 4sin(2t)$ $\Rightarrow y' = 2A\cos(2t) - 2B\sin(2t)$

Guess: $y = A\sin(2t) + B\cos(2t)$

7.) $y'' + 2y' + y = \boxed{4sin(2t) + 5cos(2t)}$

Guess: $y = A\sin(2t) + B\cos(2t)$

8.) $y'' + 2y' + y = 4sin(2t) + 5cos(3t)$

Guess for step 2a: $y = A\sin(2t) + B\cos(2t)$ $\Rightarrow f_1$

Guess for step 2b: $y = A_2\sin(3t) + B_2\cos(3t)$ $\Rightarrow f_2$

9.) $y'' + 2y' + y = \boxed{4sin(2t)} + \boxed{t+1}$

Guess for step 2a: $y = A\sin(2t) + B\cos(2t)$

Guess for step 2b: $y = At + B$

10.) $y'' + 2y' + y = (4t)(sin(2t))$ $\leftarrow$ product of degree 1 polynomial w/ a $\sin(2t)$ $\Rightarrow$ guess product

Guess: $(At+B)(C\sin(2t) + D\cos(2t))$

11.) $y'' + y = 4sin(2t)$ $\Rightarrow y = A\sin(2t) \Rightarrow y' = 2A\cos 2t \Rightarrow y'' = -4A\sin(2t)$ no y' term

Guess: $y = A\sin(2t)$

12.) $y'' + y = 4sin(t)$ $\Rightarrow y = t(A\sin t + B\cos t)$

Guess: $y = At\sin(t) + Bt\cos t$

homog $y'' + y = 0 \Rightarrow r^2 + 1 = 0 \Rightarrow r^2 = -1 \Rightarrow r = \pm i$

gen homog sol'n is $y = c_1 \sin t + c_2 \cos t$