To solve linear DE $ay'' + by' + cy = g_1 + g_2 + g_3$

Step 1: Solve homogeneous version: $ay'' + by' + cy = 0$ implies $ar^2 + br + c = 0$ implies $y = c_1\phi_1 + c_2\phi_2$.

Step 2a: Find one non-homogeneous solution, $y = f_1$, to $ay'' + by' + cy = g_1$.
Step 2b: Find one non-homogeneous solution, $y = f_2$, to $ay'' + by' + cy = g_2$.
Step 2c: Find one non-homogeneous solution, $y = f_3$, to $ay'' + by' + cy = g_3$.

Step 3: Combine all solutions to create the general solution to the non-homogeneous DE:

$$y = c_1\phi_1 + c_2\phi_2 + f_1 + f_2 + f_3$$

Last step: If IVP, plug in initial values to find the constants $c_1$ and $c_2$.

Guess a possible non-homog soln for the following DEs:

Note homogeneous solution to $y'' + 2y' + y = 0$ is $y = c_1e^{-t} + c_2te^{-t}$ since $r^2 + 2r + 1 = (r + 1)(r + 1) = 0 \Rightarrow r = -1, -1$.

1.) $y'' + 2y' + y = 4e^{2t}$

Guess: $y = Ae^{2t}$ - plug in to solve for undetermined coeff $A$

2.) $y'' + 2y' + y = 4e^t$

Guess: $y = Ae^t$

3.) $y'' + 2y' + y = 4e^{-t}$

Guess: $y = Ate^{-t}$

$y = Ae^t$ is a homog soln $\Rightarrow$ wrong guess

$y = Ate^{-t}$ is a homog soln $\Rightarrow$ wrong guess

$y = Ate^{-t}$ is a homog soln $\Rightarrow$ no soln

Multiply by $t$:

$y = Ate^{-t}$ is not homog
4.) $y'' + 2y' + y = t$
   
   Guess: $y = At + B$
   
   $y'' = A$, $y' = B$
   
   Degree 1 polynomial

5.) $y'' + 2y' + y = t + 1$
   
   Guess: $y = At + B$
   
   $y'' = A$, $y' = B$
   
   Degree 2 polynomial

6.) $y'' + 2y' + y = 4\sin(2t)$
   
   $y = A\sin(2t) + B\cos(2t)$
   
   $y' = 2A\cos(2t) - 2B\sin(2t)$

7.) $y'' + 2y' + y = 4\sin(2t) + 5\cos(2t)$
   
   $y = A\sin(2t) + B\cos(2t)$

8.) $y'' + 2y' + y = 4\sin(2t) + 5\cos(3t)$
   
   Guess for step 2a: $y = A\sin(2t) + B\cos(2t)$

   Guess for step 2b: $y = A\sin(3t) + B\cos(3t)$

9.) $y'' + 2y' + y = 4\sin(2t) + (t + 1)$
   
   Guess for step 2a: $y = A\sin(2t) + B\cos(2t)$

   Guess for step 2b: $y = At + B$

10.) $y'' + 2y' + y = 4t\sin(2t)$
    
   $\text{Product} \Rightarrow \text{guess is a product}$

   $y = (A + B)(C\sin(2t) + D\cos(2t))$

11.) $y'' + y = 4\sin(2t)$
    
   $y = A\sin(2t)$

12.) $y'' + y = 4\sin(t)$
    
   $y = At\sin(t) + Bt\cos(t)$

   $y = \frac{1}{2}(At\sin(t) + Bt\cos(t))$
\[ y = A \sin t \]

\[ \Rightarrow y' = A \cos t + At \cos t \]

\[ \Rightarrow y'' = -A \sin t + A \cos t - At \sin t \]
To solve linear DE \( ay'' + by' + cy = g_1 + g_2 + g_3 \)

**Step 1:** Solve homogeneous version: \( ay'' + by' + cy = 0 \) implies
\[
ar^2 + br + c = 0 \implies y = c_1\phi_1 + c_2\phi_2.
\]

**Step 2a:** Find one non-homogeneous solution, \( y = f_1 \), to \( ay'' + by' + cy = g_1 \)
**Step 2b:** Find one non-homogeneous solution, \( y = f_2 \), to \( ay'' + by' + cy = g_2 \)
**Step 2c:** Find one non-homogeneous solution, \( y = f_3 \), to \( ay'' + by' + cy = g_3 \)

**Step 3:** Combine all solutions to create the general solution to the non-homogeneous DE:
\[
y = c_1\phi_1 + c_2\phi_2 + f_1 + f_2 + f_3
\]

**Last step:** If IVP, plug in initial values to find the constants \( c_1 \) and \( c_2 \).

Guess a possible non-homog soln for the following DEs:

Note homogeneous solution to \( y'' + 2y' + y = 0 \) is \( y = c_1e^{-t} + c_2te^{-t} \) since \( r^2 + 2r + 1 = (r + 1)(r + 1) = 0 \)

1.) \( y'' + 2y' + y = 4e^{2t} \)
   **Guess:** \( y = Ae^{2t} \)

   
2.) \( y'' + 2y' + y = 4e^t \)
   **Guess:** \( y = Ae^t \)

3.) \( y'' + 2y' + y = 4e^{-t} \)
   **Guess:** \( y = Ae^{2t}e^{-t} \)

Since \( y = e^{-t} \) is a homog, \( y = Ae^t \) is a wrong guess

\( y = Ae^{-t} \) is also a wrong guess since it is also a homog soln

\( y = Ae^{2t}e^{-t} \) is correct guess
4.) $y'' + 2y' + y = t$

   Guess: $y = At + B$ <-- degree 1 polynomial

   $\Rightarrow$ guess is a degree 2 polynomial

5.) $y'' + 2y' + y = t + 1$

   Guess: $y = At + B$

6.) $y'' + 2y' + y = 4\sin(2t)$

   Guess: $y = A\sin(2t) + B\cos(2t)$

7.) $y'' + 2y' + y = 4\sin(2t) + 5\cos(2t)$

   Guess: $y = A\sin(2t) + B\cos(2t)$

8.) $y'' + 2y' + y = 4\sin(2t) + 5\cos(3t)$

   Guess for step 2a: $y = Asin(2t) + B\cos(2t)$

   Guess for step 2b: $y = Asin(3t) + B_2\cos(3t)$

9.) $y'' + 2y' + y = 4\sin(2t) + t + 1$

   Guess for step 2a: $y = A\sin(2t) + B\cos(2t)$

   Guess for step 2b: $y = At + B$

10.) $y'' + 2y' + y = 4\sin(2t)$

   $\Rightarrow$ product of degree 1 polynomial

   with $\sin(2t)$

   $\Rightarrow$ guess product

   Guess: $(A + B)\sin(2t) + D\cos(2t)$

11.) $y'' + y = 4\sin(2t)$

   Guess: $y = A\sin(2t)$

   $\Rightarrow$ $y'' = 4A\sin(2t)$

   $y'$ term

12.) $y'' + y = 4\sin(t)$

   Guess: $y = As\sin(\omega t) + B\cos(\omega t)$

   homog $y'' + y = 0$ $\Rightarrow$ $r^2 + 1 = 0$ $\Rightarrow$ $r^2 = -1$ $\Rightarrow$ $r = \pm i$

   gen homog soln is $y = c_1 \sin t + c_2 \cos t$