A very elementary introduction to proofs

Part 1

Example: Prove a function is 1:1

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$\Leftrightarrow$

if and only if

$f : A \to B$ is 1:1 iff $\boxed{f(x_1) = f(x_2)}$ implies $\Rightarrow$ $x_1 = x_2$.

Thus to show a function is 1:1, check if

$$\boxed{f(x_1) = f(x_2)} \text{ implies } x_1 = x_2.$$

Hypothesis: $f(x_1) = f(x_2)$. Conclusion $x_1 = x_2$.

Hypothesis implies conclusion.

$\boxed{p}$ implies q.

$p \Rightarrow q$.

Note a statement, $\boxed{p \Rightarrow q}$, is true if whenever the hypothesis p holds, then the conclusion q also holds.

To prove that a statement is true:

(1) $\boxed{\text{Assume the hypothesis holds.}}$

(2) Prove the conclusion must hold.

Ex: To prove a function is 1:1:

(1) Assume $\underline{f(x_1) = f(x_2)}$

(2) Do some algebra to prove $x_1 = x_2$.

Goal:
$x_1 = x_2$

To show a function is 1:1, check if

$$\boxed{f(x_1) = f(x_2)} \text{ implies } x_1 = x_2.$$

Example: Show $f(x) = ln(x)$ is 1:1

Proof: Suppose $f(x_1) = f(x_2)$

$\boxed{\ln(x_1) = \ln(x_2)}$

$e^{\ln(x_1)} = e^{\ln(x_2)} \Rightarrow x_1 = x_2 \quad \square$

To show a function is 1:1, check if

$$f(x_1) = f(x_2) \text{ implies } x_1 = x_2.$$

Example: Show $f(x) = ln(x)$ is 1:1

Proof:

$$ln(x_1) = ln(x_2) \;\Rightarrow\; e^{ln(x_1)} = e^{ln(x_2)} \;\Rightarrow\; x_1 = x_2.$$

To show a function is 1:1, check if

$$f(x_1) = f(x_2) \text{ implies } x_1 = x_2.$$

Example: Show $f(x) = ln(x)$ is 1:1

Proof: Suppose the hypothesis:

$\boxed{\text{Suppose } f(x_1) = f(x_2)}$

Know this you can work with this

That is, suppose $ln(x_1) = ln(x_2)$.

reword (be more specific)

Prove the conclusion holds:

Claim: $x_1 = x_2$.

$$ln(x_1) = ln(x_2) \Rightarrow e^{ln(x_1)} = e^{ln(x_2)} \Rightarrow x_1 = x_2.$$

Some notation:

$\forall$ = for all

$\exists$ = there exists

$p \Rightarrow q$

$[p \Rightarrow q]$ is equivalent to $[\boxed{\forall p,}\ q \text{ holds}]$. $\forall p, q$

That is, for everything satisfying the hypothesis p, the conclusion q must hold.

$f : A \to B$ is 1:1 iff

$$\underline{f(x_1) = f(x_2)} \text{ implies } x_1 = x_2.$$

$\Rightarrow$

$f : A \to B$ is 1:1 iff

$\boxed{\forall x_1 \text{ and } \forall x_2}$ such that $\boxed{f(x_1) = f(x_2)}$,

we have $x_1 = x_2$.