A Quick Review

of

Integration by Parts

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Integration by parts:

Derivative of a product: $(uv)' = uv' + vu'$

$$uv' + vu' = (uv)'$$

$$\int uv' = \int (uv)' - \int vu'$$

$$\int uv' = \int (uv)' - \int vu'$$

Fund thm of Calculus

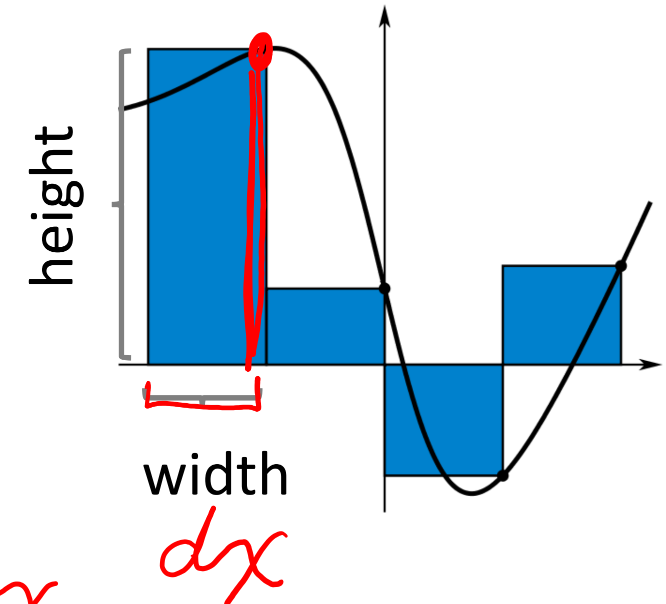
$$\boxed{\int uv' = (uv) - \int vu'}$$

Integration by parts:

Derivative of a product: $(uv)' = uv' + vu'$

$$uv' + vu' = (uv)'$$

$$uv' = (uv)' - vu'$$



$$\int uv'\,dx = \int (uv)'\,dx - \int vu'\,dx$$

$$\boxed{\int uv'\,dx = (uv) - \int vu'\,dx}$$

https://commons.wikimedia.org/wiki/File:Riemann_sum_convergence.png

Example: $\int \underset{u}{x^4} \cos(x)\, dx$

$$
\begin{aligned}
u &= x^4, & dv &= \cos(x) \\
du &= 4x^3, & v &= \sin(x)
\end{aligned}
$$

$$\int x^4 \cos(x)\, dx = x^4 \sin(x) - \int 4x^3 \sin(x)\, dx$$

Example: $\int x^4 cos(x)dx$

$$\begin{aligned} u &= x^4, & dv &= cos(x) \\ du &= 4x^3, & v &= sin(x) \\ d^2u &= 12x^2, & \int v &= -cos(x) \end{aligned}$$

$$\int x^4 cos(x) = x^4 sin(x) - \boxed{\int 4x^3 sin(x)dx}$$

$$= x^4 sin(x) - [4x^3(-cos(x)) - \int 12x^2(-cos(x))dx]$$

$$= x^4 sin(x) - 4x^3(-cos(x)) + \int 12x^2(-cos(x))dx$$

Example: $\int x^4 cos(x)dx$

$$
\begin{array}{rclcrcl}
u & = & x^4, & \quad & dv & = & cos(x) \\
du & = & 4x^3, & & v & = & sin(x) \\
d^2u & = & 12x^2, & & \int v & = & -cos(x) \\
d^3u & = & 24x, & & \int\int v & = & -sin(x)
\end{array}
$$

$$\int x^4 cos(x) = x^4 sin(x) - \int 4x^3 sin(x)dx$$

$$= x^4 sin(x) - [4x^3(-cos(x)) - \int 12x^2(-cos(x))dx]$$

$$= x^4 sin(x) - 4x^3(-cos(x)) + \int 12x^2(-cos(x))dx$$

$$= x^4 sin(x) - 4x^3(-cos(x)) + 12x^2(-sin(x))dx - \int 24x(-sin(x))dx$$

Example: $\int x^4 cos(x)dx$

$$\begin{aligned}
u &= x^4, & dv &= cos(x) \\
du &= 4x^3, & v &= sin(x) \\
d^2u &= 12x^2, & \int v &= -cos(x) \\
d^3u &= 24x, & \int\int v &= -sin(x) \\
d^4u &= 24, & \int\int\int v &= cos(x)
\end{aligned}$$

$$\int x^4 cos(x) = x^4 sin(x) - [4x^3(-cos(x))] + [12x^2(-sin(x)] - [24x cos(x)] + [24 sin(x)] - \int 0dx$$

$$\int x^4 cos(x) = x^4 sin(x) - \int 4x^3 sin(x)dx = x^4 sin(x) - [4x^3(-cos(x)) - \int 12x^2(-cos(x))dx]$$

$$= x^4 sin(x) - 4x^3(-cos(x)) + \int 12x^2(-cos(x))dx$$

$$= x^4 sin(x) - 4x^3(-cos(x)) + 12x^2(-sin(x))dx - \int 24x(-sin(x))dx$$

$$= x^4 sin(x) - 4x^3(-cos(x)) + 12x^2(-sin(x))dx - [24x(cos(x))dx - \int 24cos(x)dx]$$

$$= x^4 sin(x) - 4x^3(-cos(x)) + 12x^2(-sin(x))dx - 24x(cos(x))dx + \int 24cos(x)dx$$

Example: $\int x^4 cos(x)dx$

$$\begin{aligned} u &= x^4, & dv &= cos(x) \\ du &= 4x^3, & v &= sin(x) \\ d^2u &= 12x^2, & \int v &= -cos(x) \\ d^3u &= 24x, & \int\int v &= -sin(x) \\ d^4u &= 24, & \int\int\int v &= cos(x) \\ d^5u &= 0, & \int\int\int\int v &= sin(x) \end{aligned}$$

$$\int x^4 cos(x)dx = + x^4 sin(x) - [4x^3(-cos(x))]$$

$$+ [12x^2(-sin(x)] - [24x cos(x)]$$

$$+ [24 sin(x)] - \int 0dx$$

Example: $\int x^4 cos(x)dx$

$$\begin{aligned}
u &= x^4, & dv &= cos(x) \\
du &= 4x^3, & v &= sin(x) \\
d^2u &= 12x^2, & \int v &= -cos(x) \\
d^3u &= 24x, & \int\int v &= -sin(x) \\
d^4u &= 24, & \int\int\int v &= cos(x) \\
d^5u &= 0, & \int\int\int\int v &= sin(x)
\end{aligned}$$

$$\boxed{\begin{aligned}
\int x^4 cos(x)dx = x^4 sin(x) - [4x^3(-cos(x))] \\
+[12x^2(-sin(x)] - [24xcos(x)] \\
+[24sin(x)] + C
\end{aligned}}$$

Example: $\int e^{2x} sin(3x) dx$

Let $u = sin(3x)$, $dv = e^{2x}$

then $du = 3cos(3x)$, $v = \frac{1}{2}e^{2x}$

then $d^2u = -9sin(3x)$, $\int v = \frac{1}{4}e^{2x}$

Example: $\int e^{2x} sin(3x)dx$

Let $u = sin(3x)$, $dv = e^{2x}$

then $du = 3cos(3x)$, $v = \frac{1}{2}e^{2x}$

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$$\int e^{2x} sin(3x)dx$$

$$= \frac{1}{2}sin(3x)e^{2x} - \frac{3}{4}cos(3x)e^{2x} + \int (-\frac{9}{4})sin(3x)e^{2x}dx$$

Example: $\int e^{2x} sin(3x)dx$

Let $u = sin(3x)$, $dv = e^{2x}$

then $du = 3cos(3x)$, $v = \frac{1}{2}e^{2x}$

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$$\int e^{2x} sin(3x)dx$$

$$= \frac{1}{2}sin(3x)e^{2x} - \frac{3}{4}cos(3x)e^{2x} + \int (-\frac{9}{4}) sin(3x)e^{2x}dx$$

$$= \frac{1}{2}sin(3x)e^{2x} - \frac{3}{4}cos(3x)e^{2x} - \frac{9}{4}\int sin(3x)e^{2x}dx$$

$$\boxed{\int e^{2x} sin(3x)dx}$$

$$= \frac{1}{2} sin(3x)e^{2x} - \frac{3}{4} cos(3x)e^{2x} - \frac{9}{4} \boxed{\int e^{2x} sin(3x)dx}$$

$$\boxed{\int e^{2x} sin(3x)dx}$$

$$= \frac{1}{2}sin(3x)e^{2x} - \frac{3}{4}cos(3x)e^{2x} - \frac{9}{4}\boxed{\int e^{2x} sin(3x)dx}$$

Use algebra to solve for $\int e^{2x} sin(3x)$:

$$\int e^{2x} sin(3x)dx$$

$$= \frac{1}{2}sin(3x)e^{2x} - \frac{3}{4}cos(3x)e^{2x} - \frac{9}{4}\int e^{2x} sin(3x)dx$$

Use algebra to solve for $\int e^{2x} sin(3x)$:

$$\frac{4}{13}\ \frac{13}{4}\boxed{\int e^{2x} sin(3x)dx} = \left(\frac{1}{2}sin(3x)e^{2x} - \frac{3}{4}cos(3x)e^{2x}\right)\frac{4}{13}$$

$$\boxed{\int e^{2x} sin(3x)dx}$$

$$= \frac{1}{2} sin(3x)e^{2x} - \frac{3}{4} cos(3x)e^{2x} - \frac{9}{4} \boxed{\int e^{2x} sin(3x)dx}$$

Use algebra to solve for $\int e^{2x} sin(3x)$:

$$\frac{13}{4} \int e^{2x} sin(3x)dx = \frac{1}{2} sin(3x)e^{2x} - \frac{3}{4} cos(3x)e^{2x}$$

Thus

$$\boxed{\int e^{2x} sin(3x)dx = \frac{4}{13}[\frac{1}{2} sin(3x)e^{2x} - \frac{3}{4} cos(3x)e^{2x}]}$$