Problem Session tomorrow (Tuesday) at 5:30pm. Zoom location will be posted on ICON. announcement

Exam 2: Any 90 minute period between 1:30 - ~~3:30~~ 4pm and 5:00 - 6:30pm ← Icon e-mail

- Written Part: 2-3 questions. Please write on only 1 side. After you upload it, ask me to check it before you log out.

Proof: Induction

- Multiple Answer part: Select all correct answers. If the question is worth X points and there are N correct answers, then you earn X/N points for each correct answer and you loose X/N points for each incorrect answer (but you won't earn less than 0 on a problem.

Type questions into chat window. ← Isabel Darcy (not the e-mail

Exam 2 citations ← same as Exam 1

- Question #2
- separate place ← exam or citation

Existence and Uniqueness ← inclues NON-Linear case

Thm 2.4.2: Suppose the functions

$$z = f(t, y) \text{ and } z = \frac{\partial f}{\partial y}(t, y)$$

multivariable fns

are continuous on $(a, b) \times (c, d)$

and the point $(t_0, y_0) \in (a, b) \times (c, d)$,

$t_0 \in (a,b)$
$y_0 \in (c,d)$

then $\exists$ an interval $(t_0 - h, t_0 + h) \subset (a, b)$ such that $\exists!$ function $y = \phi(t)$ defined on $(t_0 - h, t_0 + h)$ that satisfies the following initial value problem:

IVP: $y' = f(t, y), \quad y(t_0) = y_0.$

$\exists!$ soln $\phi : (t_0 - h, t_0 + h) \to \mathbb{R}$

- includes both linear & non linear but have a stronger thm on the linear case

Theorem 4.1.1: If $p_i : (a, b) \to R$, $i = 1, ..., n$ and $g : (a, b) \to R$ are continuous and $a < t_0 < b$, then there exists a unique function $y = \phi(t)$, $\phi : (a, b) \to R$ that satisfies the initial value problem

$$y^{(n)} + p_1(t)y^{(n-1)} + ... + p_{n-1}(t)y' + p_n(t)y = g(t);$$

g is cont on (a, b)

If g is

satisfies the initial value problem

$$y^{(n)} + p_1(t)y^{(n-1)} + \dots + p_{n-1}(t)y' + p_n(t)y = g(t);$$
$$y(t_0) = y_0, \quad y'(t_0) = y_1, \dots, \quad y^{(n-1)}(t_0) = y_{n-1}$$

If g is cont $\Rightarrow$ g is cont on $(-\infty, \infty)$ which includes (a,b) $\forall a, b$

Question 4 0.5 pts

If $b^2 - 4ac < 0$ then the solution to the initial value problem $ay'' + by' + cy = 0, y(0) = -1, y'(0) = -3$ is complex valued.

- True
- False

$y'' + y = 0$

$r^2 + 1 = 0$

$\Rightarrow r = \pm\sqrt{-1} = \pm i \leftarrow$ imaginary

complex valued

But general sln

$y = c_1 \cos t + c_2 \sin t$

To find domain of sol'n take the intersectn of all the intervals

$g : (a_0, b_0) \to \mathbb{R}$ cont

$p_1 : (a_1, b_1) \to \mathbb{R}$

$\vdots$

$p_n : (a_n, b_n) \to \mathbb{R}$

Question 5 0.5 pts

If $b^2 - 4ac < 0$ then the solution to the initial value problem $ay'' + by' + cy = 0, y(0) = -1, y'(0) = -3$ is real valued.

- True
- False

when plug in to find c_i's

$\Rightarrow$ get real-valued fn

If $\exists (c,d) = \bigcap_{i=0}^{n} (a_i, b_i)$ st $t_0 \in (c,d)$

$\Rightarrow \exists!$ soln $\phi : (c,d) \to \mathbb{R}$

Roots of unity

$$(x^n)^{1/n} = (1)^{1/n} = (e^{i0})^{1/n} = \left(e^{i(0 + 2\pi k)}\right)^{1/n}$$

$$\Rightarrow \boxed{x = e^{i(2\pi k/n)}, \ k = 0, 1, 2, \dots, n-1}$$

$$\Rightarrow \boxed{x = e^{i\left(2\pi k/n\right)},\ k = 0, 1, 2, \ldots\ n-1}$$

n roots distincts

algebraic answer simplified using euler's formula

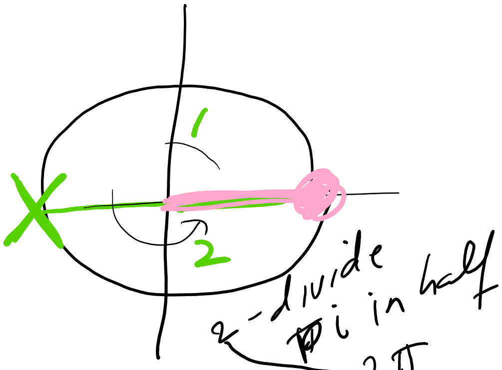
simpler

$$\boxed{x = \cos\left(\frac{2\pi k}{n}\right) + i\sin\left(\frac{2\pi k}{n}\right) \quad k = 0, 1, \ldots, n-1}$$

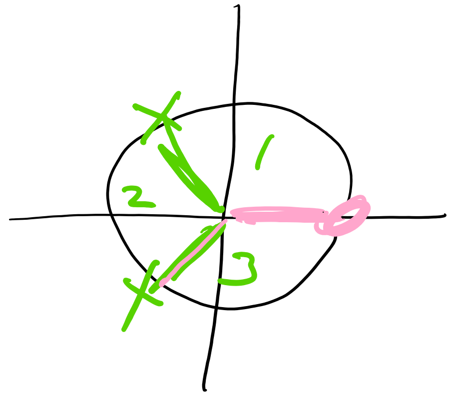
If you check answer, cite source (like last exam)

But you must show work for credit!!!

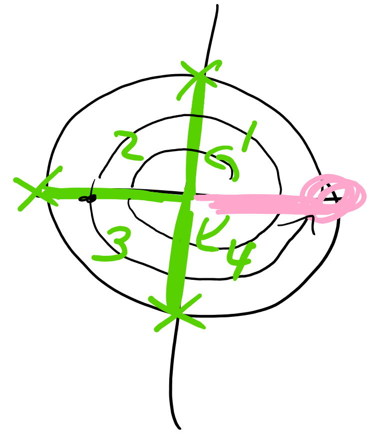
Be able to do it both algebraically and graphically (& explain graphically)



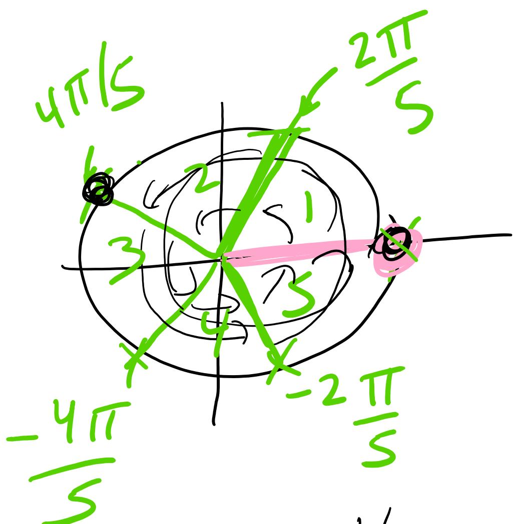
$\sqrt{1} = 1^{1/2}$



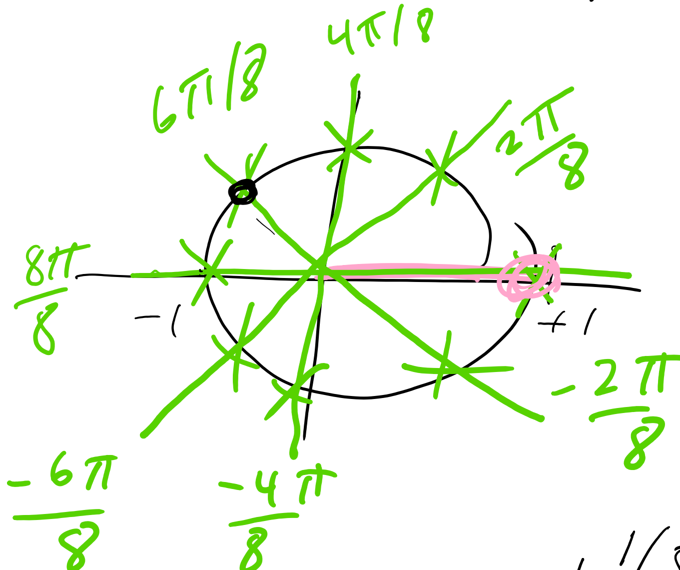
$1^{1/3}$



$1^{1/4}$



$y = 1^{1/5}$



$y = 1^{1/8}$

$$y^n = -1 \Rightarrow y = (-1)^{1/n}$$

$$(-1)^{1/n} = \left(e^{i\pi}\right)^{1/n} = \left(e^{i(\pi + 2\pi k)}\right)^{1/n}$$

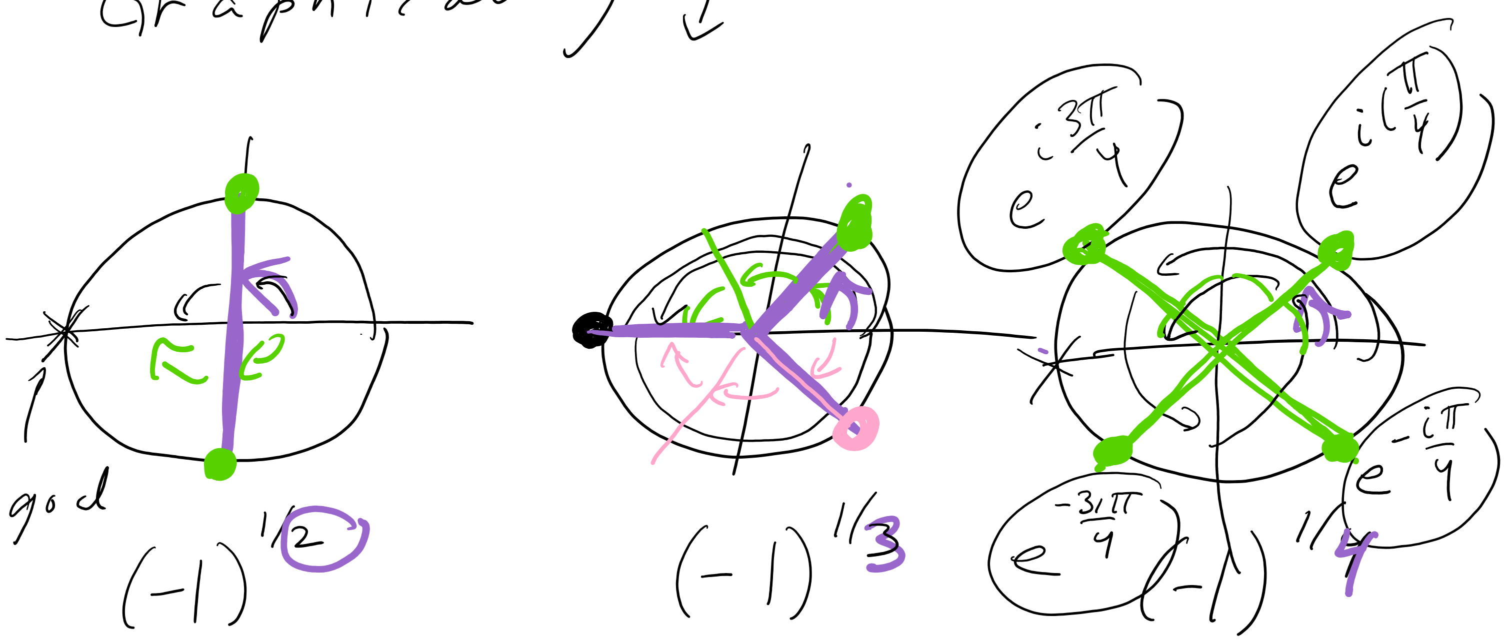
$$\boxed{(-1)^{\frac{1}{n}} = e^{i\left(\frac{\pi + 2\pi k}{n}\right)}, \quad k = 0, \ldots, n-1}$$

↓ simplify using Euler's formula

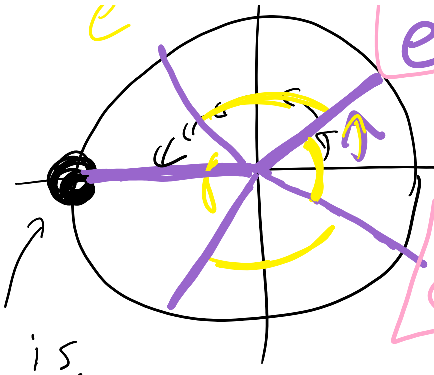
$$\boxed{(-1)^{\frac{1}{n}} = \cos\left(\frac{\pi + 2\pi k}{n}\right) + i\sin\left(\frac{\pi + 2\pi k}{n}\right), \quad k = 0, \ldots, n-1}$$

Algebraically ↗

Graphically ↓

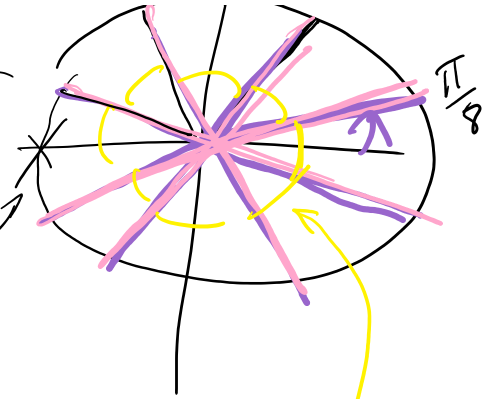


$e^{\cdots s}$



$e^{-\frac{i\pi}{4}}$

-1 is a soln to $\rightarrow (-1)^{1/5}$



-1 is not a soln $\rightarrow (-1)^{1/8}$

all yellow $\frac{2\pi}{8}$

soln's will always be $\rightarrow$ complex conjugate pairs

4.3) # 13

$$y^{(4)} + 2y''' + 2y'' = 3e^{t} + 2te^{-t} + e^{-t}\sin t$$

Guess nonhomog:

Step 1: Find <u>homog</u>

(since this can affect our nonhomog guess)

$$r^4 + 2r^3 + 2r^2 = r^2(r^2 + 2r + 2) = 0$$

$$= 0$$

$$r = 0, 0, \quad r = \frac{-2 \pm \sqrt{4 - 4(2)}}{2} = \frac{-2 \pm \sqrt{-4}}{2}$$

$$= -1 \pm i$$

NON gen homog soln: $y = c_1(1) + c_2(t) + c_3(e^{-t}\cos t) + c_4 e^{-t}\sin t) + \psi$

where $\psi = ?$

$$\underline{y^{(4)} - 2y''' - 2y''} = 3e^{t} + 2te^{-t} + \underbrace{e^{-t}\sin t}_{\text{homog}}$$

not hom, not homog

NOTE TYPO

$$= \underbrace{(3+2t)e^{-t}} + \underbrace{e^{-t}\sin t}$$

$\psi_1 = (A+Bt)e^{-t}$ since no y term and no y' term

$\psi_1' = (B)e^{-t} - (A+Bt)e^{-t}$

$\psi_1'' = -Be^{-t} - [Be^{-t} - (A+Bt)e^{-t}]$

$\psi^{(4)} + 2\psi''' + 2\psi'' = (3+2t)e^{-t}$

$$y^{(4)} + 2y''' + 2y'' = (3+2t)e^{-t}$$

when plug in e^{-t} term will cancel, leaving me with degree 1 poly $= (3+2t)$

Because of product rule

Guess for $\psi_1 = (A+Bt)e^{-t}$

$\psi_2 =$

plug in and get $\underbrace{e^{-t}\sin t}$ but this is homog

$$\psi_2 = e^{-t}(A\sin t + B\cos t)t$$

$y^{(4)} - 2y'''$