Exam 1 Wednesday over 2.1, 2.4, 2.5, 2.8 (including induction), ch 3, 4, 7.1 – 7.5

7.2: 17, 19
7.3: 13, 14, 15
7.1 (use matrix form): 3, 4, 5, 6, 12
7.5: 1b, 2b, 5b

Problem session Monday during class.

Problem session Tuesday ??? (submit survey by Sunday night if you wish to help choose the time).

HW 8, ungraded survey due Sunday night.
Real quiz 3 due Monday night.
In class: All Quizzes 10/7 - 10/21 due Monday night.
HW 9 knowledge due Wednesday (but can turn in Friday – Sunday)

Quiz Instructions

WHILE TAKING THIS QUIZ: Post links to any web resources that you use for this quiz to the pinned ICON discussion for Quiz 3. Note you should post the full URL.

For example, if you use WolframAlpha to compute 1+1:
Incorrect post: https://www.wolframalpha.com/
Correct Post: https://www.wolframalpha.com/input/?i=1%2B1

Please include the problem #.

If the site requires payment and/or registration, please state so.

Note: you have an unlimited number of attempts.

Solve $\mathbf{x}' = \begin{pmatrix} 3 & 1 \\ 5 & 0 \end{pmatrix} \mathbf{x}$

Step 1: Find eigenvalues:

$$\begin{vmatrix} 3-r & 1 \\ 5 & 0-r \end{vmatrix} = (3-r)(-r) - 5 = r^2 - 3r - 5 = 0$$

$$\text{Thus } r = \frac{3 \pm \sqrt{9-4(1)(-5)}}{2} = \frac{3 \pm \sqrt{29}}{2}$$

Step 2: Find eigenvectors:

$$\begin{pmatrix} 3 - (\frac{3 \pm \sqrt{29}}{2}) & 1 \\ 5 & 0 - (\frac{3 \pm \sqrt{29}}{2}) \end{pmatrix} \mathbf{v} = \mathbf{0}$$

Eigenvalues: $r = \frac{3 \pm \sqrt{29}}{2}$

$$\begin{pmatrix} \frac{3 \mp \sqrt{29}}{2}) & 1 \\ 5 & \frac{-3 \mp \sqrt{29}}{2}) \end{pmatrix} \begin{bmatrix} \quad \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

General solution:

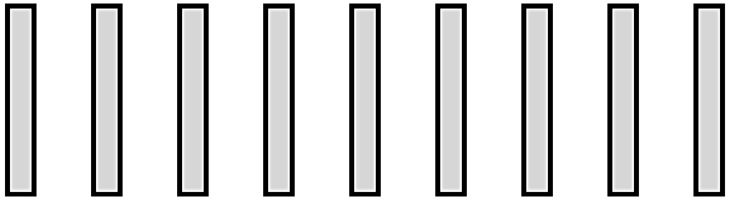
Eigenvalues: $r = \frac{3 \pm \sqrt{29}}{2}$

$$\begin{pmatrix} \frac{3 \mp \sqrt{29}}{2}) & 1 \\ 5 & \frac{-3 \mp \sqrt{29}}{2}) \end{pmatrix} \begin{bmatrix} \qquad \\ \qquad \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

General solution:

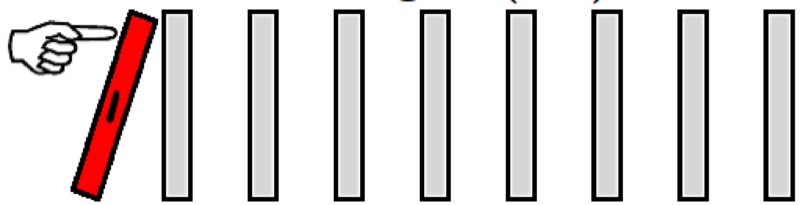
Domino effect of Induction

VU University Amsterdam
Faculty of Sciences
Department of Computer Sciences



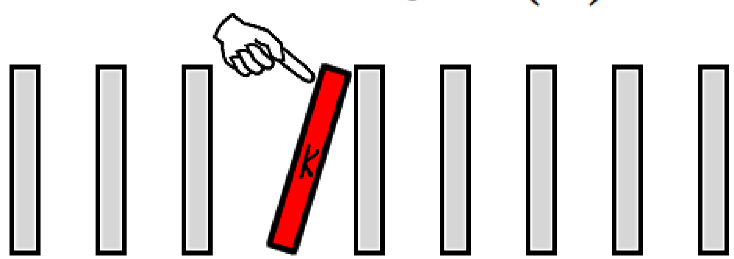
We have a statement $S(n)$ that we need to prove.

Base case: Proving $S(n_0)$ is like knocking down the first domino in the row:

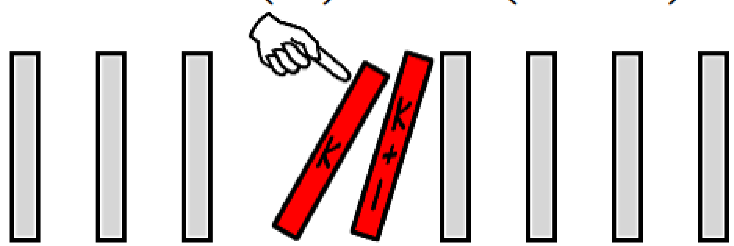


Show $S(n_0)$ is true.

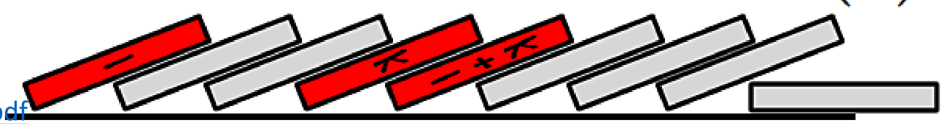
Hypothesis: Assume for some $k \geq n_0$, $S(k)$ holds



Induction Step: Show that $S(k) \Rightarrow S(k+1)$



If we do all of the above, all the dominoes fall: $S(n)$ holds!



pics courtesy: Coolmath Algebra

http://www.few.vu.nl/~rbakhshi/teaching/induction-handout.pdf http://www.few.vu.nl/~rbakhshi/teaching/induction-handout.pdf

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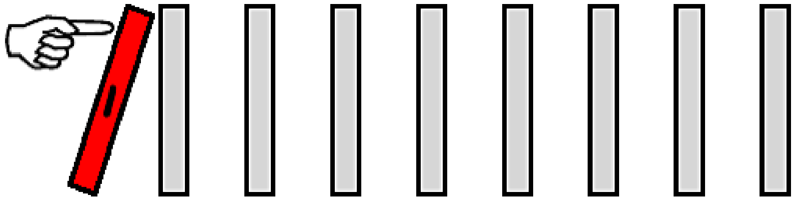

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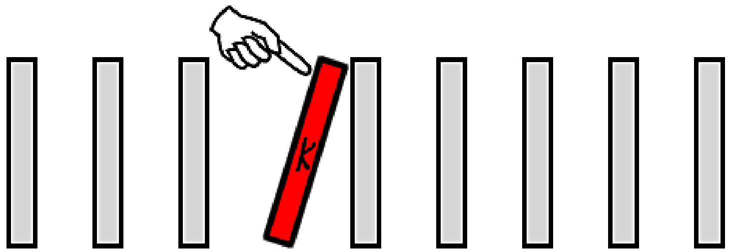

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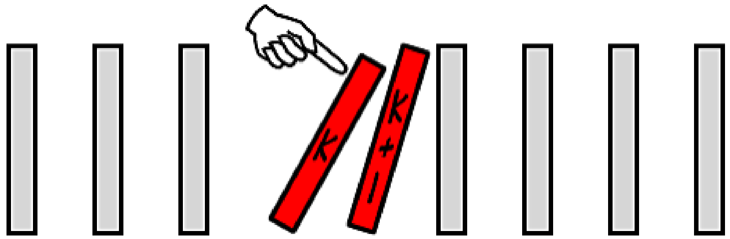

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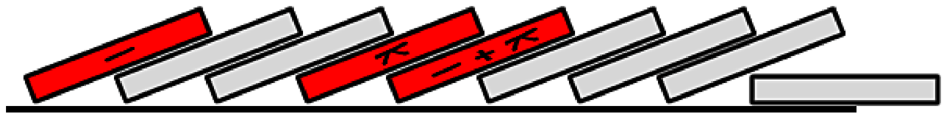

□

pics courtesy: Coolmath Algebra

Show by induction that if

$$f(t, y) = 2t(1 + y),$$

$$\phi_{n+1}(t) = \int_0^t f(s, \phi_n(s))ds,$$

and $\phi_0(t) = 0,$

then $\phi_n(t) = \displaystyle\sum_{k=1}^{n} \frac{t^{2k}}{k!}$ for $n \geq 1$

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Proof by induction on n:

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Proof by induction on n:

For $n = 1$,

$$\phi_1(t) = \int_0^t f(s, \phi_0(s))ds = \int_0^t f(s, 0)ds$$
$$= \int_0^t 2s(1+0)ds = s^2|_0^t = t^2$$

Show by induction that if $f(t, y) = 2t(1 + y)$,

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$$\sum_{k=1}^{1} \frac{t^{2k}}{k!} = \frac{t^{2(1)}}{(1)!} = t^2 = \phi_1(t)$$

Show by induction that if $f(t,y) = 2t(1+y)$,

$$\phi_0(t) = 0, \text{ and } \phi_{n+1}(t) = \int_0^t f(s, \phi_n(s))ds,$$

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Proof by induction on n.

$n = 1$: Claim: $\phi_1(t) = \displaystyle\sum_{k=1}^{1} \frac{t^{2k}}{k!}$

$$\phi_1(t) = \int_0^t f(s, \phi_0(s))ds = \int_0^t f(s, 0)ds$$

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Induction hypothesis:

Show by induction that if $f(t, y) = 2t(1 + y)$,

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Induction hypothesis:

Suppose for $n = j - 1$, $\quad \phi_{j-1}(t) = \sum_{k=1}^{j-1} \frac{t^{2k}}{k!}$

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Suppose for $n = j-1$, $\phi_{j-1}(t) = \sum_{k=1}^{j-1} \frac{t^{2k}}{k!}$

Hypothesis: $f(t,y) = 2t(1+y)$, $\phi_0(t) = 0$,

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Proof of claim: $\phi_j = \int_0^t f(s, \phi_{j-1}(s))ds$

$$= \int_0^t 2s(1 + \phi_{j-1}(s))ds$$

$$= \int_0^t 2s(1 + \sum_{k=1}^{j-1} \frac{s^{2k}}{k!})ds$$

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