What is one of the most important concepts that we have covered? **Enter your answer(s) in the chat box.**

Poll to be taken near end of class: What time(s) do you prefer for a Tuesday problem session?

11am, 12:30pm, 4:30pm, 6pm

Exam 2 will be similar to quiz 2, but timed (90 minutes).

$\leq 60\%$ $\geq 40\%$

- Exam will include multiple choice as well as a written part that you should upload to ICON
- You should also upload your citations (either with the problem or separately).
- You can start anytime between 1:30pm and 2:30pm.
- You have 90 minutes.
- You can use the chat to text me questions anytime during the exam.
- I will post messages to slide if there are multiple similar questions.
- **Do NOT log off until I have checked your upload**. ← chat me when finished

citations Th night

Discussion

Free online textbooks:

RESOURCES

Isabel Darcy

Paul's Online Notes: https://tutorial.math.lamar.edu/classes/de/de.aspx

Elementary Differential Equations by William Trench

https://www.oercommons.org/courses/elementary-differential-equations-with-boundary-value-problems-2/view

?

Notes of Diffy Qs: Differential Equations for Engineers by Jiri Lebl: http://www.jirka.org/diffyqs/

Draw direction field: ~~https://c3d.libretexts.org/DirectionField/index.html~~

https://c3d.libretexts.org/DirectionField/index.html

add 1 more

Videos:

www.integralcalculator.com

Joshua Pankau's Math 2560 lectures

Engineering Math

For more resources:

on exam show work

https://www.oercommons.org/hubs/open-textbooks

https://www.geogebra.org/search/differential%20equations (later)

Pinned Discussions

RESOURCES

All Sections

Resources for exam 1 material

0 4

1 Section

Last post at Sep 21 at 11:39am

To do Sep 26 at 11:59pm

HW/Quiz Answers

0 3

All Sections

Last post at Sep 21 at 12:12pm

HW/Quiz Answers

Nicholas Connolly

All Sections

on this page

Solutions to selected HW and quiz problems.

Anyone can request specific problems as well.

You may upload citations to "Exam 1 citations", type them in here, or include in your scanned in upload, or any combination of these 3 (plus e-mail). Please be specific. For example, for problem 1, I used *** to do *** and *** to do ***. Cite your review sheet

(1) type in citations here

For partial credit on multiple choice answers, you can also type in your alternative answers here.

(2) upload to exam 1 citation (due Thursday)
+ review sheet

(3) In written part of exam

HTML Editor

B I U A A Tx x² x₂

12pt

last question on exam + alt answers

What substitution should be made to transform the following equation into a linear equation: $yy' = y^2 + t^5 y^6$

a.) $v = t^4$

b.) $v = t^5$

c.) $v = t^6$

d.) $v = t^{-4}$

e.) $v = t^{-5}$

f.) $v = t^{-6}$

g.) $v = y^4$

h.) $v = y^5$

i.) $v = y^6$

j.) $v = y^{-4}$

k.) $v = y^{-5}$

l.) $v = y^{-6}$

a.) t ^ 4

b

1V

$1y' + p(t)y = g(t)$

opts

$\frac{yy'}{y^6} - \frac{y^2}{y^6} = \frac{t^5 y^6}{y^6}$

$\boxed{y' y^{-5}} - \boxed{y^{-4}} = t^5$

g(t)

y^{-4}

$v' = 4y^3 y'$

$v' = -4y^{-5}y'$

Let $v = y^{-4} \Rightarrow \boxed{v' = -4y^{-5}y'}$

$-\frac{1}{4}v' - v = t^5$

$\boxed{1v'} + p(t)\boxed{v} = t^5$

3.) Circle the differential equation whose direction field is given below: $f(t, y)$

A) ~~$y' = t^2$~~

B) ~~$y' = \frac{1}{2}t + 1$~~

C) ~~$y' = e^t$~~

D) ~~$y' = t + 1$~~

E) ~~$y' = 2e^t$~~

F) $y' = y - t$

G) ~~$y' = ln(t)$~~

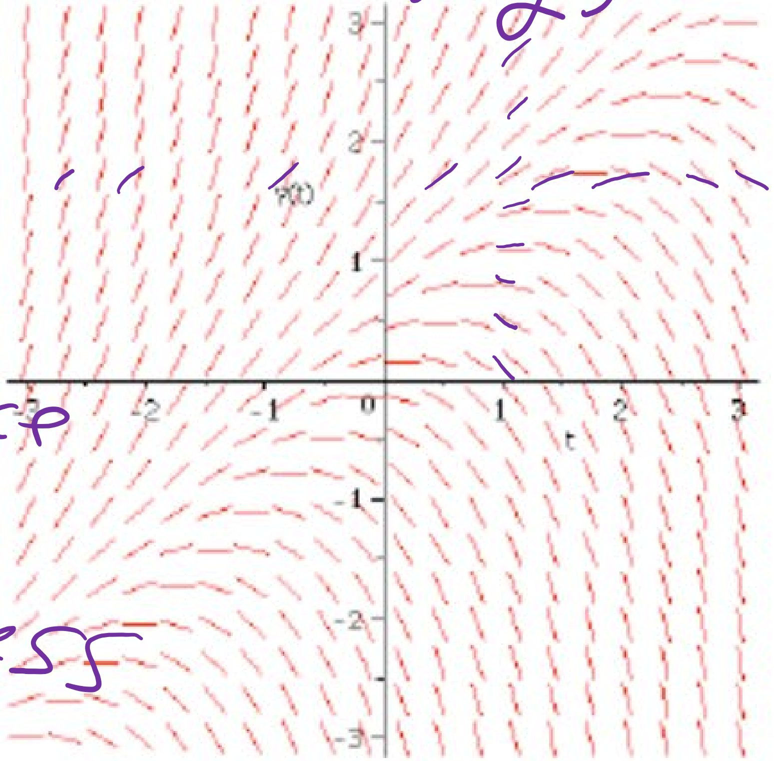
H) ~~$y' = 0$~~

I) $y' = sin(t)$

J) $y' = cos(t)$

① slope fields

② existance & uniquness



2.) Circle a solution to the differential equation whose direction field is given below:

A) $\cancel{y = t^2}$

B) $y = \frac{1}{2}t + 1$

C) $\cancel{y = e^t}$

D) $y = t + 1$

E) $\cancel{y = -2e^t}$

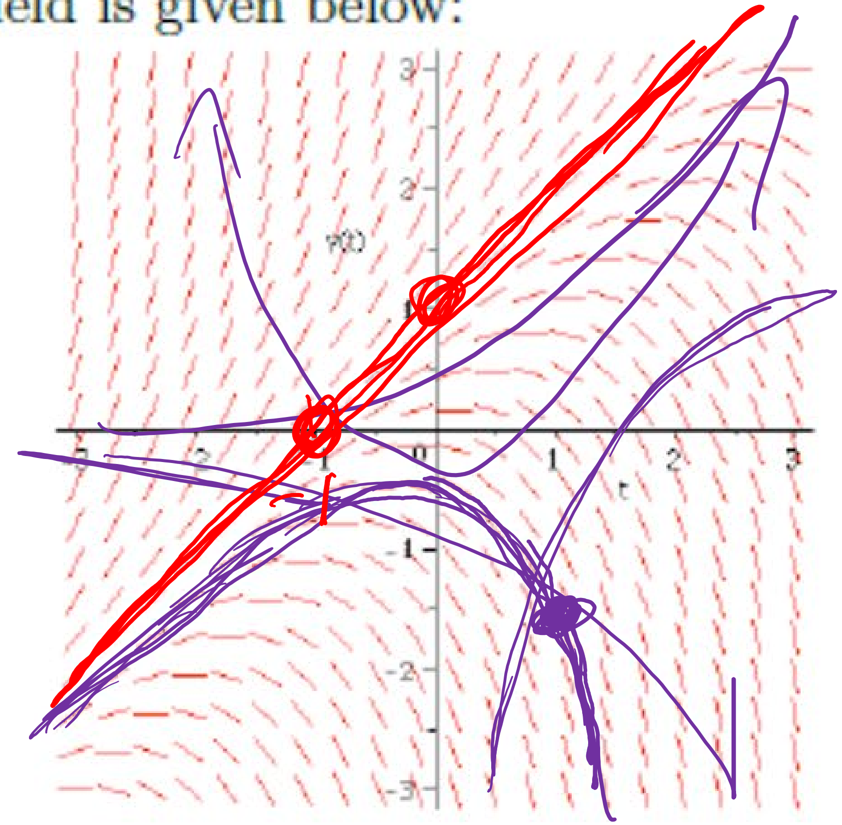
F) $y = 2t + 1$

G) $\cancel{y = ln(t)}$

H) $\cancel{y = 0}$

I) $\cancel{y = sin(t)}$

J) $\cancel{y = cos(t)}$



asym Stable, unstable & semi-stable of Equil soln

4.) Circle the general solution to the differential equation whose direction field is given below:

$t \to -\infty$

$y \to -\infty$

A) $\cancel{y = t + C}$ B) $\cancel{y = t^2 + C}$

C) $\cancel{y = e^t + C}$ D) $y = Ce^t + t + 1$

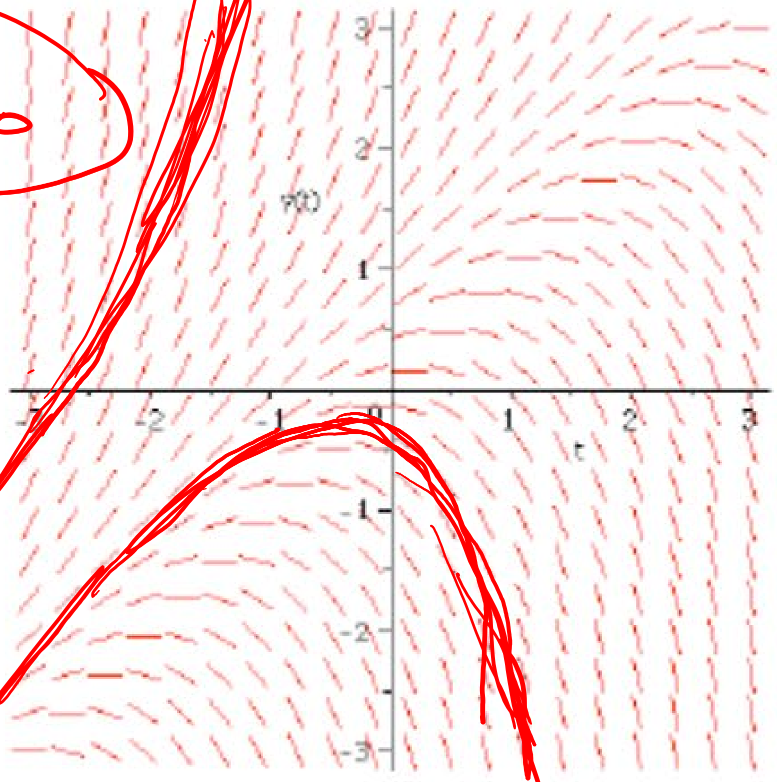
E) $\cancel{y = Ce^t}$ $t \to -\infty$ F) $\cancel{y = e^t + t + C}$

$Ce^t \to 0$

G) $\cancel{y = \ln(t) + C}$ H) $\cancel{y = C}$

I) $\cancel{y = \sin(t) + C}$ J) $\cancel{y = \cos(t) + C}$

no equil soln



$f(t) + C$ ✗

$\lim_{t \to +\infty}$

Solving DE

1rst order DE

linear

$\underbrace{3}_{3} y' + \underbrace{3}_{3} y = \underbrace{g}_{3}(t)$

$1y' + p(t)\,y = G(t)$

$\vdots$

$\boxed{\text{FTC}}$

product rule

$\underbrace{\int (u\,y)'\,dt} = \underbrace{\int \sim\, dt}$

not linear

Separable

alg + calc 1

$\boxed{\dfrac{dv}{dt} = \dfrac{dv}{dx}\dfrac{dx}{dt}}$

Bernoulli

~~d~~ transform to linear

Existance $\emptyset\emptyset$

&

uniqueness

compare linear Thm 2.4.1

can find domain

Thm 2.4.2

Approx 5 ch

2 - 8

$\boxed{\cancel{12:30m} \;\&\; 4:30pm}$

12:30pm 6pm

Make-up exam time: Wednesday 5pm – 6:30pm.

Office hr sol'ns

p101 #35 $\qquad y'' + (y')^2 = 2e^{-y}$

Let $v(t) = y'(t) \Longrightarrow v' = y''$

$$v = \frac{dy}{dt} \Longrightarrow \frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d^2y}{dt^2} = y''$$

$$\cancel{\frac{dv}{dy} = \frac{d}{dy}\left(\frac{dy}{dt}\right) \neq y''}$$

When to use chain rule

Let $v(x) = y'(t(x))$

$$\frac{dv}{dx} = \frac{d}{dx}\left(\frac{dy}{dt}\right) = \frac{d}{dx}\left(y'(t(x))\right)$$

$$= y''(t(x)) \cdot t'(x)$$

Bad use of variables since different than previous choice

$$\frac{d}{dt}\left(y^{-4}\right) = -4y^{-5}\frac{dy}{dt} = -4y^{-5}y'$$

$y(t)$

$$\frac{d}{dt}\left(t^{-4}\right) = -4t^{-5}\underbrace{\frac{dt}{dt}}_{t'=1} = -4t^{-5}$$

$$\frac{dv}{dt} = \frac{dv}{\cancel{dy}} \cdot \frac{\cancel{dy}}{dt}$$

$$\underline{y(t) = \sin(t)}$$

$$\frac{d}{dt}\left[(\sin(t))^{-4}\right] =$$

$$-4(\sin t)^{-5} \frac{d}{dt}(\sin(t))$$

$$\frac{d}{dt}(v(y)) = dv$$

$v(y) = y^{-4}$

$$\frac{d}{dt}(y^{-4}) = (-4y^{-5}) \cdot \frac{dy}{dt}$$

$$\frac{dv}{dy} \cdot \frac{dy}{dt}$$

$$\underbrace{\frac{dv}{dt}}_{} = \frac{dv}{dy} \cdot \left(\frac{dy}{dt}\right) = \underbrace{(v) \cdot \frac{dv}{dy}}_{}$$

chain rule

where $v = y' = \frac{dy}{dt}$ ✓

p101 #35

$$y'' + (y')^2 = 2e^{-y}$$

Let $v = y' \Rightarrow v' = y''$

$v' = \frac{dv}{dt}$

where $\boxed{v = \frac{dy}{dt}}$

$$\rightarrow \frac{dv}{dt}\cancel{v'} + v^2 = 2e^{-y}$$

$$\frac{dv}{dt} + v^2 = 2e^{-y}$$

$$\frac{dv}{dt} = \frac{dv}{dy}\frac{dy}{dt} = \frac{dv}{dy}\cdot v$$

$$v\frac{dv}{dy} + v^2 = 2e^{-y}$$

$$v\frac{dv}{dy} + v^2 = 2e^{-y}$$

not linear & not separable

Bernoulli = not in our class

<u>NOT</u> in general

or if hint given

Transform it into

$$1z' + p(y)z = g(y)$$

$$\boxed{\left(v\frac{dv}{dy}\right) + v^2 = 2e^{-y}}$$

Let $z = v^2 \Rightarrow \dfrac{dz}{dy} = 2v \cdot \dfrac{dv}{dy}$

$$\frac{1}{2}\left(\frac{dz}{dy} + (z)\right) = 2e^{-y} \leftarrow \text{linear}$$

$$1\frac{dz}{dy} + 2z = 4e^{-y} \overset{\text{work}}{\Longrightarrow} z =$$

$$v^2 =$$

$$\left(\frac{dy}{dt}\right)^2 =$$

#36) $y' \, y'' = 2$

Let $v = y' = \frac{dy}{dt} \Rightarrow \frac{dv}{dt} = v' = y''$

$$v v' = 2$$

Linear & separable → $v \frac{dv}{dt} = 2 \Rightarrow \int v \, dv = \int 2 \, dt$

choice of method

$$\frac{1}{2} v^2 = 2t + C$$

$$\frac{1}{2} (y')^2 = 2t + C$$

$$\frac{1}{2}(y'(t))^2 = 2t + C$$

Use now or at end

$y'(0) = 2$: $\frac{1}{2}(2)^2 = 2(0) + C$

$$\Rightarrow C = 2$$

$$(y'(t))^2 = 4t + 4$$

$$y'(t) = \cancel{\pm\sqrt{4t+4}}$$

$$+\sqrt{4t+4}$$

$\vdots$

But $y'(0)=2$
so only need $+$

continue to find y

#37 $(1+t^2) \cdot v' + 2tv + 3t^2 = 0$

where $v' = \frac{dv}{dt}$

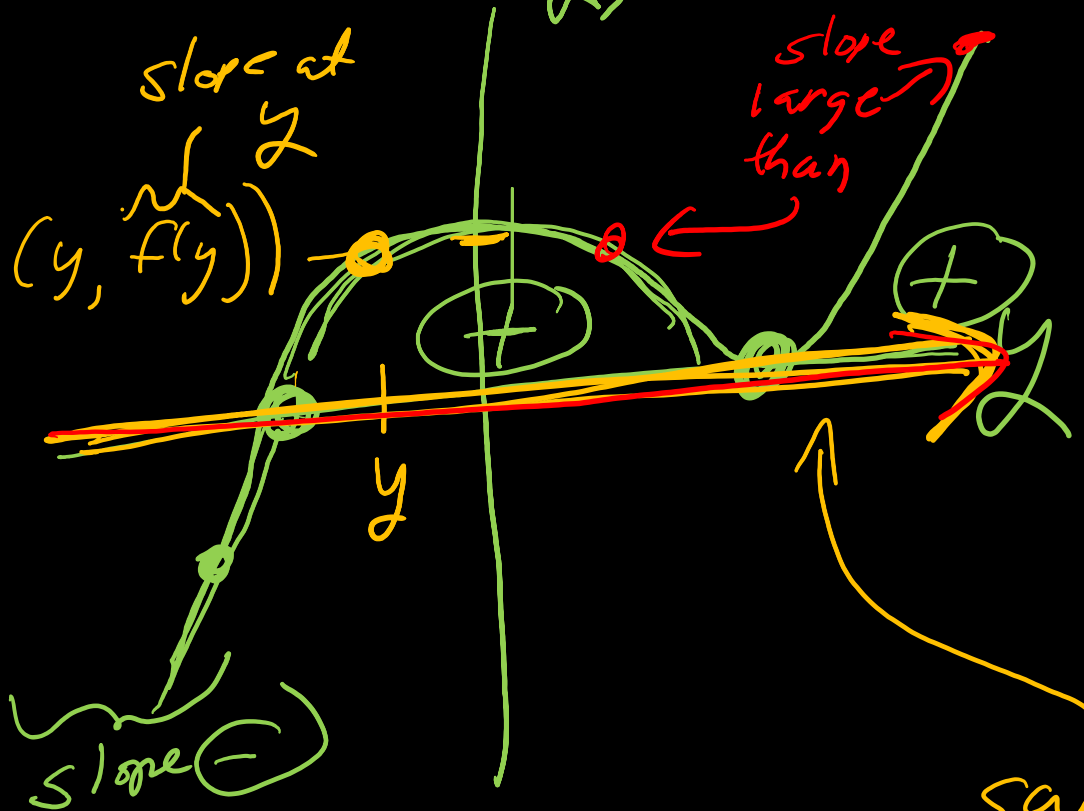
Linear, not separable.

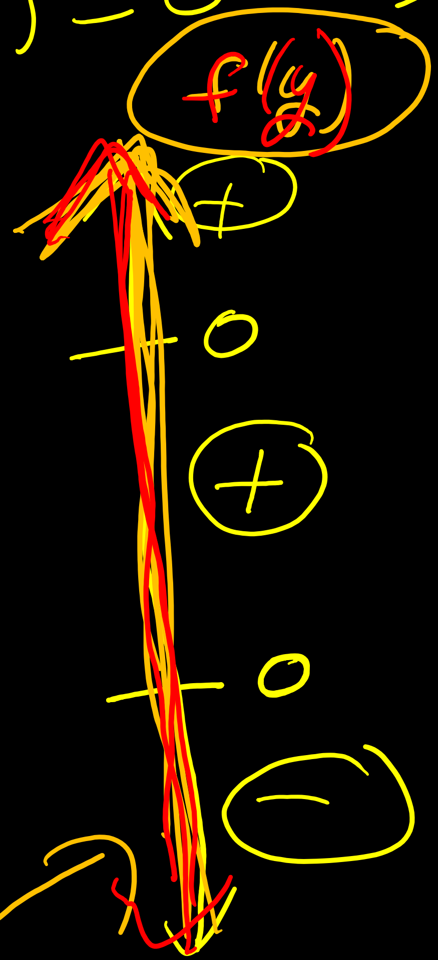
so use integrating factor

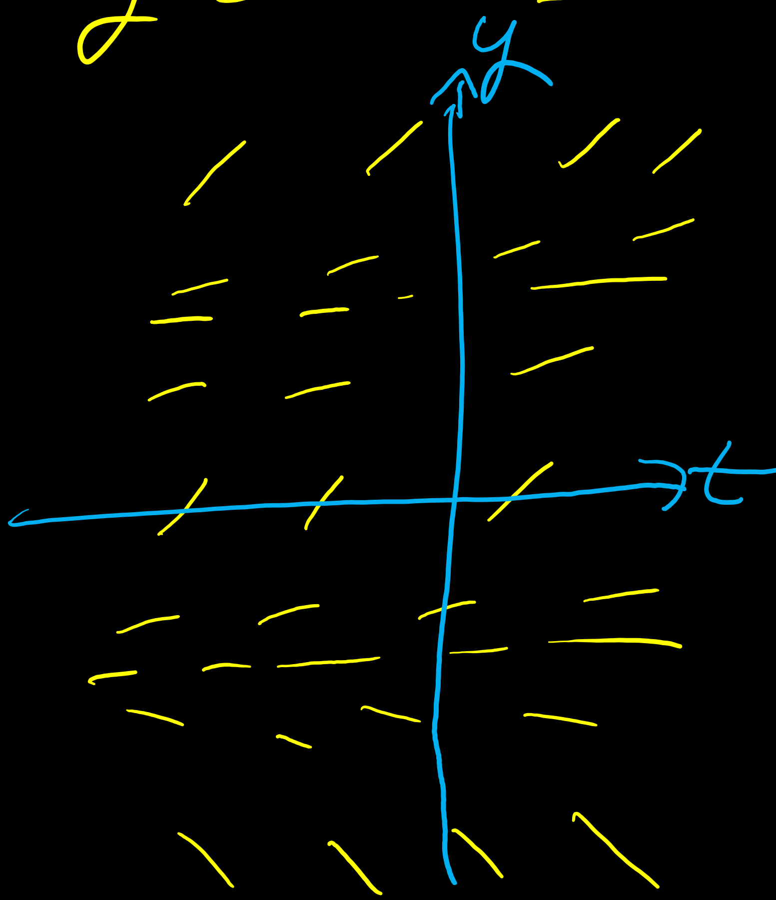
2.5 $y' = f(y)$ Autonomous

To find equil solns: $f(y) = 0 \Rightarrow y = \_\_\_$

$f(y) = \text{slope}$

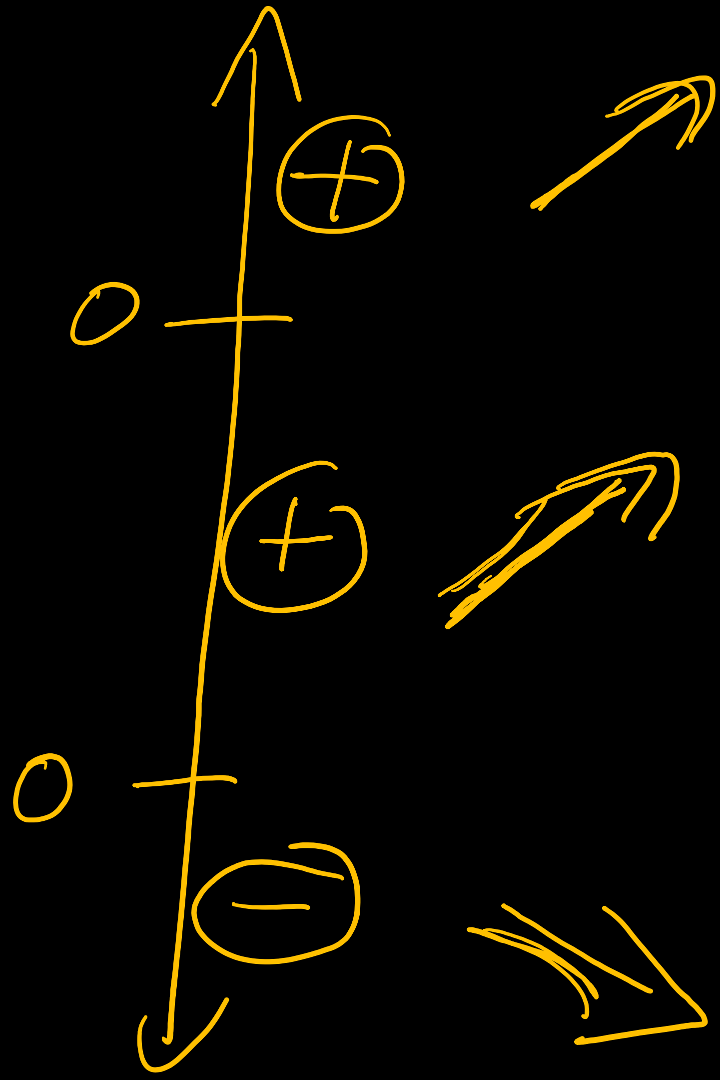






slope field in (t y) plane

same rotate 90°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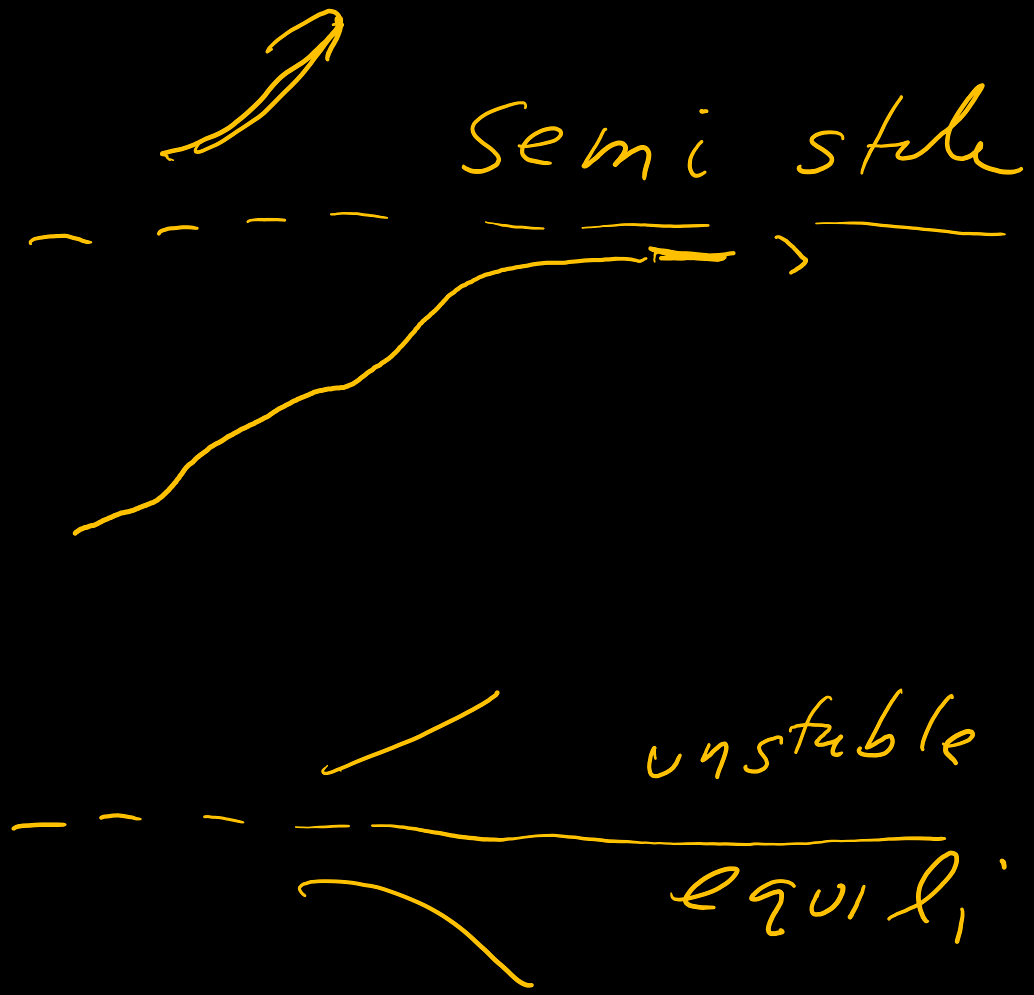
semi stble
unstable
equili

asym stable

0

$-$

asym stable

$+$

$\leftarrow y = A$

$$\lim_{t \to \infty} y(t) = A$$

Falling Ball

$$F = ma = m\frac{dv}{dt}$$

$$F = F_g + F_{\text{air resistance}}$$

$$F = +mg - v^2$$ for falling ball

+

$F_{air} = v^2$

Ball thrown up

$$F = mg + v^2$$

F_{air}

$F_{air} = v$

Falling ball

F_{air}

$v > 0$

if positive direction down

Thrown up

F_{air}

$\boxed{v < 0}$

if positive direction is down

$F = mg + F_{air} = mg - v$