In Class Quizzes Part 1

- In Class Quizzes prior to Midterm 1 on September 23 (ICQs between 8/24 and 9/21), worth 14 points total.
- Note that the highest grade on this assignment is 100%.
- If your score on this assignment is less than 100%, you can still bring your score up to 100%.

In Class Quizzes Part 2 (10/7 - Week 13)

Bonus points: All quizzes 10/7 – 10/21 , All quizzes ? - ?, Final exam all quizzes 8/24 - ??, Posting on ICON discussion page, chats from last Wednesday,

Talk about math and game theory in political science

The AWM (Association for Women in Mathematics) student chapter is hosting a talk this Thursday, October 22 from 3:30 to 4:30 via Zoom (see link below). Dr. Elizabeth Menninga, Assistant Professor in the Department of Political Science, will be talking about how her studies in math led her to research in political science.

This talk will be geared toward undergraduates.

Zoom Meeting Details:
https://uiowa.zoom.us/j/98706452660?pwd=c3ZsUUkzV2FWTW1uV3FoOXBjOG9tQT09

Meeting ID: 987 0645 2660
Passcode: 119820

Quiz 3
due Sunday
3pts w/ unlimited

post citations
on discussion
page DURING quiz

Definitions: Ch 3, 4, 7

Linear combination: of $\{y^{(n)}, \ldots, y\}$

$$p_0(t)\, y^{(n)} + p_1(t)y^{(n-1)} + \ldots + p_{n-1}(t)y' + p_n(t)y$$

Linear DE: diff Equation

$$y^{(n)} + p_1(t)y^{(n-1)} + \ldots + p_{n-1}(t)y' + p_n(t)y = g(t)$$

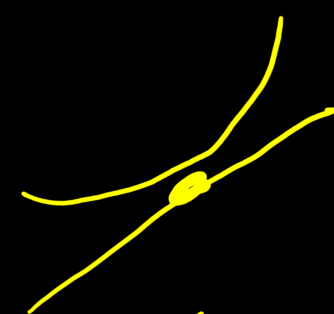
Homogeneous DE:

$$y^{(n)} + p_1(t)y^{(n-1)} + \ldots + p_{n-1}(t)y' + p_n(t)y = 0$$

IVP: n^{th} order

$$y^{(n)} + p_1(t)y^{(n-1)} + \ldots + p_{n-1}(t)y' + p_n(t)y = g(t),$$

$$y(t_0) = y_0,\ y'(t_0) = y_1, \ldots, y^{(n-1)}(t_0) = y_{n-1} \leftarrow n \text{ values}$$



Suppose $a_0 y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1} y' + a_n y = g(t)$

n l.i slns to span soln space.

has solution $y = c_1\phi_1(t) + \ldots + c_n\phi_n(t) + \psi(t)$ Guess $y = e^{rt}$

Characteristic polynomial what we use to find homog soln charecn

$$a_0 r^n + a_1 r^{n-1} + \ldots + a_n = 0$$

Fundamental set of solutions of homog eqn

$\{\phi_1, \ldots, \phi_n\}$ if ϕ_i are l.i so that we have a basis

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Wronskian = determinant of the coefficient matrix needed to solve for c_i in IVP

Wronskian in ch 4 (includes ch 3) $n=2$

$$\det\begin{bmatrix} \phi_1(t) & \cdots & \phi_n(t) \\ \phi_1'(t) & \cdots & \phi_n'(t) \\ \vdots & & \vdots \\ \phi_1^{(n-1)}(t) & \cdots & \phi_n^{(n-1)}(t) \end{bmatrix}$$

take derivative of the n l.i solns to homo eqn to create $\underline{n \times n}$ matrix

Wronskian = det of coef matr

2 x 2 case

$y(t_0) = y_0 \; : \; y_0 = c_1 \phi_1(t_0) + c_2 \phi_2(t_0) + \psi(t_0)$

$y'(t_0) = y_1 \; : \; y_1 = c_1 \phi_1'(t_0) + c_2 \phi_2'(t_0) + \psi'(t_0)$

in matrix form

$$\begin{bmatrix} y_0 \\ y_1 \end{bmatrix} = \begin{bmatrix} \phi_1(t_0) & \phi_2(t_0) \\ \phi_1'(t_0) & \phi_2'(t_0) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \psi(t_0) \\ \psi'(t_0) \end{bmatrix}$$

This has ! soln for c_1 & c_2

$\iff \det W(\phi_1, \phi_2)(t_0) \neq 0$

7.1: Transforming an n^{th} order linear DE into a system of n first order linear DEs.

Ex: $y'''' - 5y'' + 6y = \sin(t)$

4th order $\Rightarrow$ 4 first order linear DE w/ 4 unknowns $x_1\ x_2\ x_3\ x_4$

derivat

$y = x_1$

$y' = x_2 = x_1'$

$y'' = x_3 = x_2'$

$y''' = \boxed{x_4} = x_3$

$y'''' = -6y + 5y'' + \sin(t)$

$= -6x_1 + 5x_3 + \sin$

$y'''' = x_4' = -6x_1 + 5x_3 + \sin(t)$

$$y'''' - 5y'' + 6y = \sin(t) \Rightarrow y''' = -6y + 5y'' + \sin t$$

in matrix form

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -6 & 0 & 5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sin(t) \end{bmatrix}$$

$$y = x_1$$

$$y' = x_1' = x_2$$

$$y'' = x_2' = x_3$$

$$y''' = x_3' = x_4$$

$$y'''' = x_4' = -6x_1 + 5x_3 + \sin(t)$$

in non matrix form

7.4 - 7.6, 9.1

7.4 7.4-7.6, 9.1

Solve the homogeneous linear DE: $\vec{\mathbf{x}}' - A\vec{\mathbf{x}} = \mathbf{0}$

A is a square matrix, $\vec{X}$ is a vector $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

$X' = AX$.

Educated Guess

$X = \vec{v}e^{rt} \Rightarrow X' = r\vec{v}e^{rt}$

Plug in: $r\vec{v}\cancel{e^{rt}} = A\vec{v}\cancel{e^{rt}}$ $\quad e^{rt} \neq 0$

$r\vec{v} = A\vec{v}$ or equiv $A\vec{v} = r\vec{v}$

7.4 - 7.6, 9.1

Solve the homogeneous linear DE: $\mathbf{x}' - A\mathbf{x} = \mathbf{0}$

$\mathbf{x}' = A\mathbf{x}$ Guess $x = \mathbf{v}e^{rt}$. Plug in to find $\mathbf{v}$ and r:

$$[\mathbf{v}e^{rt}]' = A\mathbf{v}e^{rt} \quad \text{implies} \quad r\mathbf{v}e^{rt} = A\mathbf{v}e^{rt} \quad \text{implies} \quad r\mathbf{v} = A\mathbf{v}.$$

Thus $\underbrace{\mathbf{v} \text{ is an eigenvector}}$ with $\underbrace{\text{eigenvalue } r.}$

To solve $x' = A\vec{v}$

find e. values of A

and their corresponding e. vectors

Note since the equation is homogeneous and linear,

Linear Hom syst of DE

linear combinations of solutions are also solutions:

Suppose $\mathbf{x} = \mathbf{f_1}(t)$ and $\mathbf{x} = \mathbf{f_2}(t)$ are solutions to $\mathbf{x}' = A\mathbf{x}$.

prove by plugging in

Then $\mathbf{f_1}' = A\mathbf{f_1}$ and $\mathbf{f_2}' = A\mathbf{f_2}$

Thus $[c_1\mathbf{f_1} + c_2\mathbf{f_2}]' = c_1\mathbf{f_1}' + c_2\mathbf{f_2}' = c_1 A\mathbf{f_1} + c_2 A\mathbf{f_2} = A(c_1\mathbf{f_1} + c_2\mathbf{f_2})$.

Alt proof: Note $L(\vec{x}) = x' - A\vec{x}$

is a linea fn so

$L(c_1 f_1 + c_2 f_2) = c_1 L(f_1) + c_2 L(f_2)$

Suppose an object moves in the 2D plane (the x_1, x_2 plane) so that it is at the point $(x_1(t), x_2(t))$ at time t. Suppose the object's velocity is given by

convert to matrix form

$$\boxed{\begin{aligned} x_1'(t) &= 4x_1 + x_2, \\ x_2'(t) &= 5x_1 \end{aligned}}$$

e. vector

e. value

Or in matrix form $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 4 & 1 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Guess $x = \mathbf{v}e^{rt}$

$x = ve^{\lambda t}$

To solve, find eigenvalues and corresponding eigenvectors:

$$\begin{vmatrix} 4-r & 1 \\ 5 & -r \end{vmatrix} = (4-r)(-r) - 5 = r^2 - 4r - 5 = (r-5)(r+1).$$

Thus $r = -1, 5$ are eigenvalues.

use r of λ

Eigenvectors associated to eigenvalue $r = -1$: $\begin{pmatrix} 5 & 1 \\ 5 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{1}{5} \\ 0 & 0 \end{pmatrix}$

Thus x_2 is free and $x_1 + \frac{1}{5}x_2 = 0$

Hence the eigenspace corresponding to $r = -1$ is

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{5}x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} -\frac{1}{5} \\ 1 \end{pmatrix}$$

Thus $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ is an eigenvector with eigenvalue $r = -1$

$\begin{pmatrix} 2 \\ -10 \end{pmatrix} e^{-t}$.

Hence $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix} e^{-t}$ is a solution.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 4^{+1} & 1 \\ 5 & 0^{+1} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\boxed{\text{Guess } x = \mathbf{v}e^{rt}}$$

To solve, find eigenvalues and corresponding eigenvectors:

$$\begin{vmatrix} 4-r & 1 \\ 5 & -r \end{vmatrix} = (4-r)(-r) - 5 = r^2 - 4r - 5 = (r-5)(r+1).$$

Thus $r = -1, 5$ are eigenvalues.

Find nullspace of $\begin{bmatrix} 5 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$$

E. vectors associated to e. value $r = 5$: $\begin{pmatrix} -1 & 1 \\ 5 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Thus $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector with eigenvalue $r = 5$ since it is a nonzero solution to the above equation.

Hence $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t}$ is also a solution.

Note since the equation is homogeneous and linear,

linear combinations of solutions are also solutions:

Suppose $\mathbf{x} = \mathbf{f_1}(t)$ and $\mathbf{x} = \mathbf{f_2}(t)$ are solutions to $\mathbf{x}' = A\mathbf{x}$.

Then $\mathbf{f_1}' = A\mathbf{f_1}$ and $\mathbf{f_2}' = A\mathbf{f_2}$

Thus $[c_1\mathbf{f_1} + c_2\mathbf{f_2}]' = c_1\mathbf{f_1}' + c_2\mathbf{f_2}' = c_1A\mathbf{f_1} + c_2A\mathbf{f_2} = A(c_1\mathbf{f_1} + c_2\mathbf{f_2})$.

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vector format

Hence the general solutions is $\boxed{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = c_1 \begin{pmatrix} -1 \\ 5 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t}}$

Or in non-matrix form: $x_1(t) = -c_1e^{-t} + c_2e^{5t}$ 1rst row

$x_2(t) = 5c_1e^{-t} + c_2e^{5t}$ 2nd row

$$\vec{x}' = \begin{pmatrix} 3 & -1 \\ 5 & 0 \end{pmatrix} \vec{x}$$

1. Find e. values

$$\begin{vmatrix} 3-r & 1 \\ 5 & 0-r \end{vmatrix} = (3-r)(-r) - 5$$

$$= r^2 - 3r - 5 = 0$$

$$r = \frac{3 \pm \sqrt{9 - 4(1)(-5)}}{2} = \frac{3 \pm \sqrt{29}}{2}$$

$$\begin{pmatrix} 3 & 1 \\ 5 & 0 \end{pmatrix}$$

$$r = \frac{3-\sqrt{29}}{2}$$

Step 2 Find e. vectors

$$\frac{6}{2}$$

$$\begin{bmatrix} \cancel{3}-\left(\frac{3-\sqrt{29}}{2}\right) & 1 \\ 5 & 0-\left(\frac{3-\sqrt{29}}{2}\right) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$$

$$\begin{bmatrix} \left(\frac{3}{2} + \frac{\sqrt{29}}{2}\right) & 1 \\ 5 & \left(-\frac{3+\sqrt{29}}{2}\right) \end{bmatrix} \begin{bmatrix} -1 \\ \left(\frac{3}{2} + \frac{\sqrt{29}}{2}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 \\ \frac{3}{2} + \frac{\sqrt{29}}{2} \end{bmatrix}}_{\text{e. vector for e. value } \frac{3-\sqrt{29}}{2}}$$

If r_1 is an e.value of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} \boxed{a - r_1 \quad b} \\ \boxed{c \quad d - r_1} \end{pmatrix}$$

these rows will be multiples of each other

Rows are lin <u>dependent</u>
iff $\exists$ non zero soln