Create an example $y' = f(\cancel{t}, y)$ such that $y = g(t)$ is a solution to this D.E. and $g(0) = 2$, but $g(t) \to \infty$ as $t \to \infty$.

Suppose y = 0 is a stable equilibrium solution for the differential equation y' = f(t, y). If y = g(t) is a solution to the initial value problem

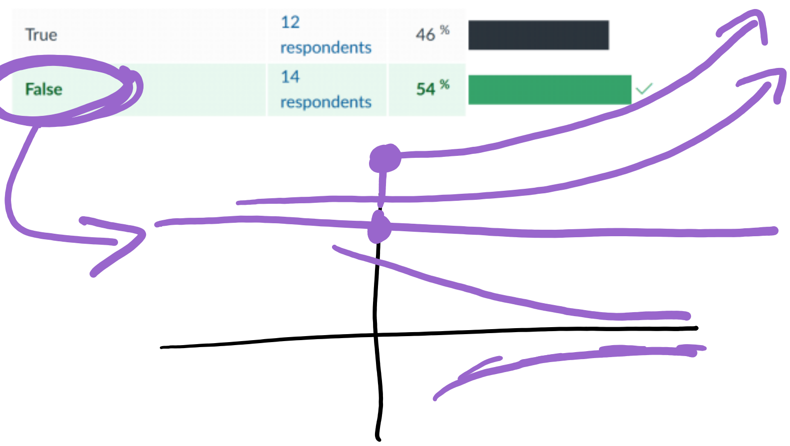
$$y' = f(t, y), \quad y(0) = 2$$

Then $lim_{t\to\infty} g(t) = 0$ X

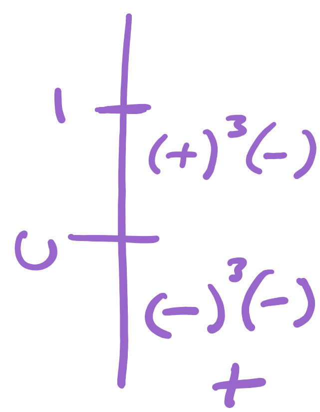
"obviously" false

Hint: Draw direction field with initial value (0, 2).

True	12 respondents	46 %	
False	14 respondents	54 %	✓



$$y' = y^3(\underline{y-1})$$



$$\lim_{t\to\infty} y(y-1) = ??$$

$$\lim_{t\to\infty} f(t, y) = ? \quad \text{DNE}$$

since don't know y

$$\lim_{t\to\infty} f(t, 0) = \lim_{t\to\infty} 0$$

$$\lim_{t\to\infty} f(t,\cdot) = \lim_{t\to\infty} 0$$

since $\begin{cases} y = 1 \\ y = 0 \end{cases}$ is an equilibrium soln

$$y' = f(t,y) = 0$$

$$\lim_{t\to\infty} f(t,1) = 0$$

since slope $y = 1$ is zero

MC = 55pts

Average = 46pts

Monday: 2.8 Pf by induction

Today: Ch 3

Tonight: post HW 5 due Sunday

Ch 3: 2nd order LINEAR eqns

Let's look at some relevant Ch 2 problems first

Focus on constant coefficient 2nd order linear homogeneous

$$ay + by' + cy = 0$$

linear comb $= 0$ $\Leftarrow$ homog

3.1 – 3.4

EX of 1rst order linea

$\boxed{\text{linear and separable}}$

$$y' + ay = 0, \quad a \in \mathbb{R} \quad \cancel{a \neq 0}$$

$$\frac{dy}{dt} = -ay$$

$$\frac{1}{a}\int \frac{dy}{\cancel{a}y} = -\int dt$$

$$\frac{1}{a}\ln|ay| = -t + C$$

check $\left[\frac{1}{a}(\ln|ay|)' = \frac{1}{a}\frac{1}{ay}\cdot a = \frac{1}{ay}\right]$

$$e^{\ln|ay|} = e^{-at + C}$$

$$ay = Ce^{-at}$$

$$y = Ce^{-at}$$

2nd order linear homog ex from ch 2

$$\boxed{u'' + au' = 0}$$

from ch 2

$$\Rightarrow \boxed{y'' + ay' = 0}$$

Let $v = y' \Rightarrow v' = y''$

$$\Rightarrow v' + av = 0$$

From last example $v = Ce^{-at}$

$$y' = Ce^{-at}$$

$$\frac{dy}{dt} = Ce^{-at}$$

$$\int dy = \int Ce^{-at}\,dt$$

$$\boxed{y = C_1 e^{-at} + C_2}$$

2nd order $\Rightarrow$ <u>expect</u> 2 constants

true for linear case

$$\boxed{\text{General soln to } y'' + ay' = 0}$$

Educated Guess & Check

$$ay'' + by' + cy = 0$$

Guess: $y = e^{rt}$

Guess: $y = c$

$\Rightarrow y' = re^{rt} \quad y'' = r^2 e^{rt}$

Check (Plug in):

$$a r^2 \underline{e}^{rt} + b r \underline{e}^{rt} + c \underline{e}^{rt} = 0$$

$$e^{rt}(ar^2 + br + c) = 0$$

$e^{rt} \neq 0$ So can divide both sides by e^{rt} w/o loosing solns

$$ar^2 + br + c = 0$$

$$\Rightarrow r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If 2 different roots
ie $b^2 - 4ac \neq 0$

$\Rightarrow r = r_1 \text{ \& } r_2$

$\Rightarrow y = e^{r_1 t}$ and $y = e^{r_2 t}$ are both solns

From linear algebra it is "obvious" that the general soln is

$^{r_1 t}$ $^{r_2 t}$

$$\boxed{y = c_1 \underbrace{e^{r_1 t}} + c_2 \underbrace{e^{r_2 t}}}$$

EX: $y'' + ay' = 0$

Gues $y = e^{rt}$

$y' = re^{rt}$

$y'' = r^2 e^{rt}$

$$r^2 \cancel{e^{rt}} + ar\cancel{e^{rt}} = 0$$

$$r^2 + ar = 0$$

$$r(r+a) = 0$$

$$\Rightarrow r = 0, \quad r = -a$$

$$\Rightarrow \underbrace{y = e^{\underline{0}t}}_{y=1} \quad \text{and} \quad y = e^{\underline{-a}t}$$

Linear combination $\Rightarrow$ general soln

$$\boxed{y = c_1(1) + c_2 e^{-at}}$$

general soln

Ex: $y'' - 5y' + 6y = 0$

$$y = e^{rt} \Rightarrow y' = re^{rt} \Rightarrow y'' = r^2 e^{rt}$$

no primes
no r's in front
of $y = e^{rt}$

$$r^2 \cancel{e^{rt}} - 5r\cancel{e^{rt}} + 6\cancel{e^{rt}} = 0$$

$$\underline{1}y'' - \underline{5}y' + 6 = 0 \Rightarrow \underline{r^2} - 5r + 6 = 0$$

$$(r-3)(r-2) = 0 \Rightarrow \begin{matrix} r=2 \\ r=3 \end{matrix}$$

$\Rightarrow y = e^{3t}$ & $y = e^{2t}$ are soln

General soln is

$$y = c_1 e^{2t} + c_2 e^{3t}$$

$$y'' + \underline{y} = 0$$

Educated Guess $y = e^{rt}$ & plug in

$$r^2 + \underline{1} = 0$$

not $\cancel{r^2 + r = 0}$

$$y'' + y = 0 \Rightarrow r^2 e^{rt} + e^{rt} = 0$$

$$r^2+1=0$$

$$\Rightarrow r^2=-1 \Rightarrow r=\pm\sqrt{-1}$$

$$\Rightarrow r=\pm i$$

$$\Rightarrow y=e^{it} \text{ and } y=e^{-it}$$

are solutions

Real

when taking linear comb to solve real IVP

Note this is a 3.3 problem

$$y=e^{it}=\cos t+i\sin t$$

$$y=e^{-it}=\cos t-i\sin t$$

$$y=\frac{e^{it}+e^{-it}}{2}=\frac{2\cos t}{2}=\cos t$$

$$\Rightarrow y=\cos t \text{ is a soln}$$

$$y=-i\left(\frac{e^{it}-e^{-it}}{2}\right)=-i\left(\frac{2i\sin t}{2}\right)$$

$$=-\frac{\cancel{2}}{\cancel{2}}i^2\sin t=\sin t$$

$$\boxed{y=c_1\cos t+c_2\sin t}$$

$$\boxed{\begin{array}{c} y = c_1 \cos t + c_2 \sin t \\ \text{general soln} \end{array}}$$

3.3 $\quad ar^2 + br + c = 0$

$$r = u \pm iv$$

$\Rightarrow$ general soln is

$$y = c_1 e^{ut} \cos(vt) + c_2 e^{ut} \sin(vt)$$

3.4: repeated root

$$r = r_1$$

$$y = c_1 e^{r_1 t} + c_2 t e^{r_1 t}$$

Def: A function f is linear if

$\rightarrow f(ax) = af(x)$

$\rightarrow f(x+y) = f(x) + f(y)$

. . .

Ex: $y = \ln t$ is not linear