Each assignment worth 10 points

A. 7 points for completion,

B. 3 points for the chosen graded problem: section 1.1 #10

   The chosen problem to be graded should be on the first page and clearly identified (with a box, high-lighted, etc.).

Use a low but readable resolution (high resolutions take too long to open in ICON). Can include several pages in one image.

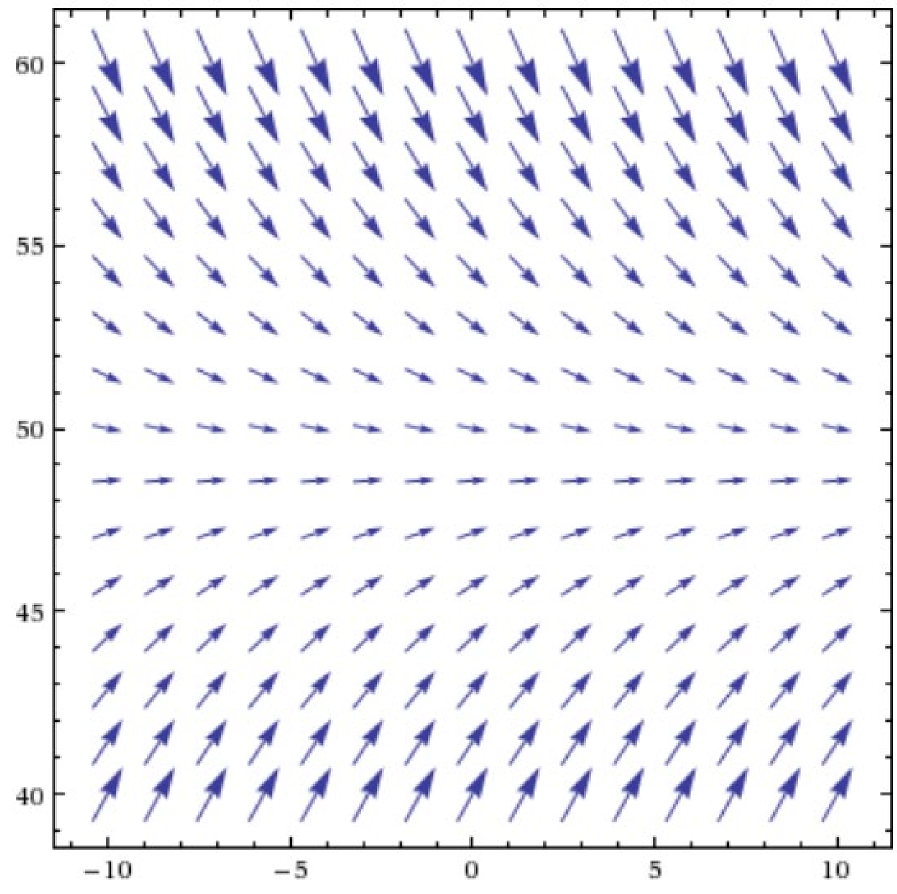
Assignments officially due on Fridays at 11:59 PM, but can still be submitted late through Sunday at 11:59.

Late policy: -3 points per day, allowing for minimum of 4 points as long as something reasonable is uploaded by Sunday even if incomplete.

File uploads restricted to .pdf files (other file types may not be compatible with ICON).

1.1: Examples of differentiable equation:

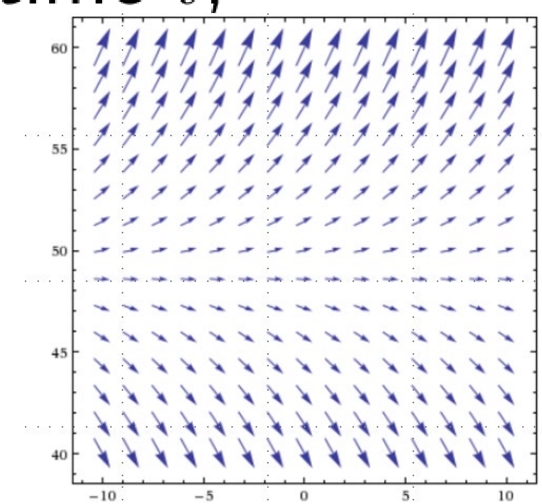
1.) Ball example: $F = ma = m\frac{dv}{dt} = mg - \gamma v$



2.) Mouse population increases at a rate proportional to the current population:

Model : $\frac{dp}{dt} = rp - k$
where $p(t)$ = mouse population at time t,
r = growth rate or rate constant,
k = predation rate = # mice killed per unit time.

3.) Continuous compounding $\frac{dS}{dt} = rS + k$
where $S(t)$ = amount of money at time t,
r = interest rate,
k = constant deposit rate



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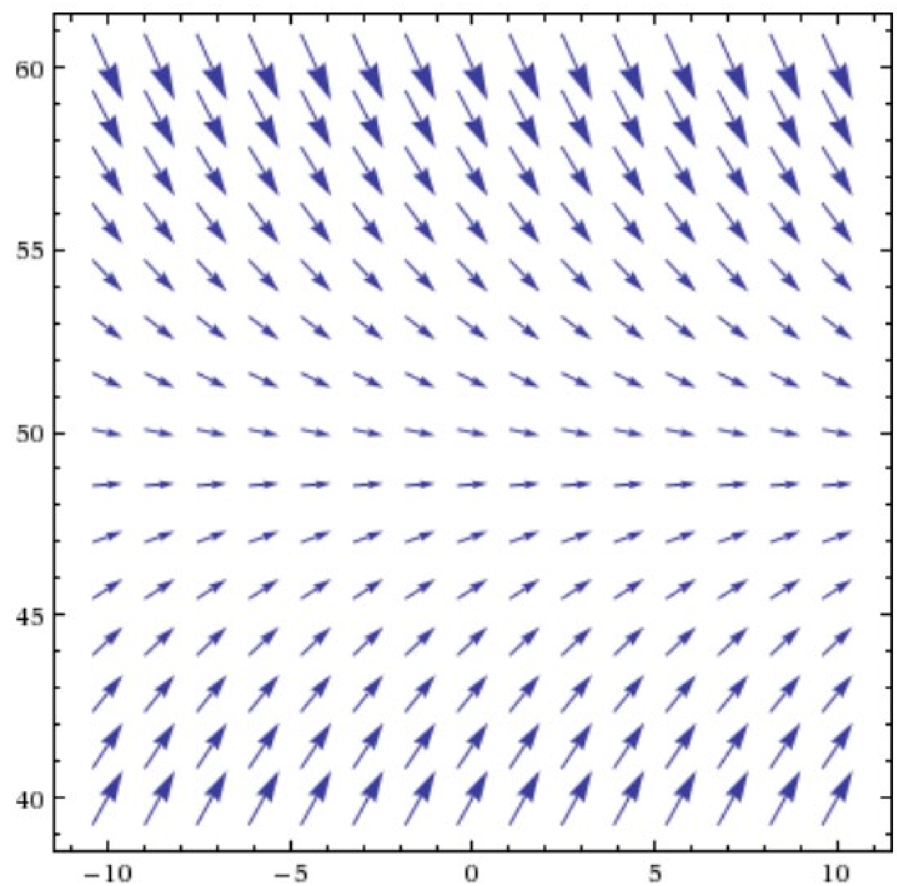

F_g = Gravitational force $= -mg$

IF the positive direction points up.

Note in some examples in the book, the positive direction points down ($F_g = +mg$) while in other examples in the book, the positive direction points up ($F_g = -mg$)

2.3: Modeling with differential equations.

Ex.: $F = ma = mv' = m\frac{dv}{dt}$

$x =$ position

$v =$ velocity $= x'$

$a =$ acceleration $= v' = x''$

$t =$ independent variable,

$x, v, a =$ dependent variables

$m =$ mass

$mg =$ weight

Calc 1 review

Model 1: Falling ball near earth, neglect air resistance.

$F_g =$ Gravitational force $= -mg$

$mv' = -mg$ implies $v' = -g$.

$$\frac{dv}{dt} = g \implies dv = gdt$$

Thus $v = -gt + C$.

IVP: $v(0) = v_0$:

$v_0 = -g(0) + C$ implies $C = v_0$. Thus $v = -gt + v_0$

Calc 1 review (continued)

$x' = v = -gt + v_0$ implies $x = -\frac{1}{2}gt^2 + v_0 t + C$.

IVP: $x(0) = x_0$:

$$x_0 = -\frac{1}{2}g(0)^2 + v_0(0) + C \text{ implies } C = x_0.$$

Thus $x = -\frac{1}{2}gt^2 + v_0 t + x_0$.

Note $v = 0$ when ball reaches maximum height.

If ball is dropped (as opposed to thrown up or down), then $v(0) = 0$.

Differential equations (improved model)

Model 2: Falling ball near earth, include air resistance.

Let $A(v) =$ the force due to air resistance.

$mv' = F_g + R(v) = -mg + A(v)$

Example from section 1.1 (but with positive direction pointing up:

If air resistance proportional to velocity, then $A(v) = \gamma v$

If $\gamma \geq 0$, then $mv' = F_g + R(v) = -mg - \gamma v$

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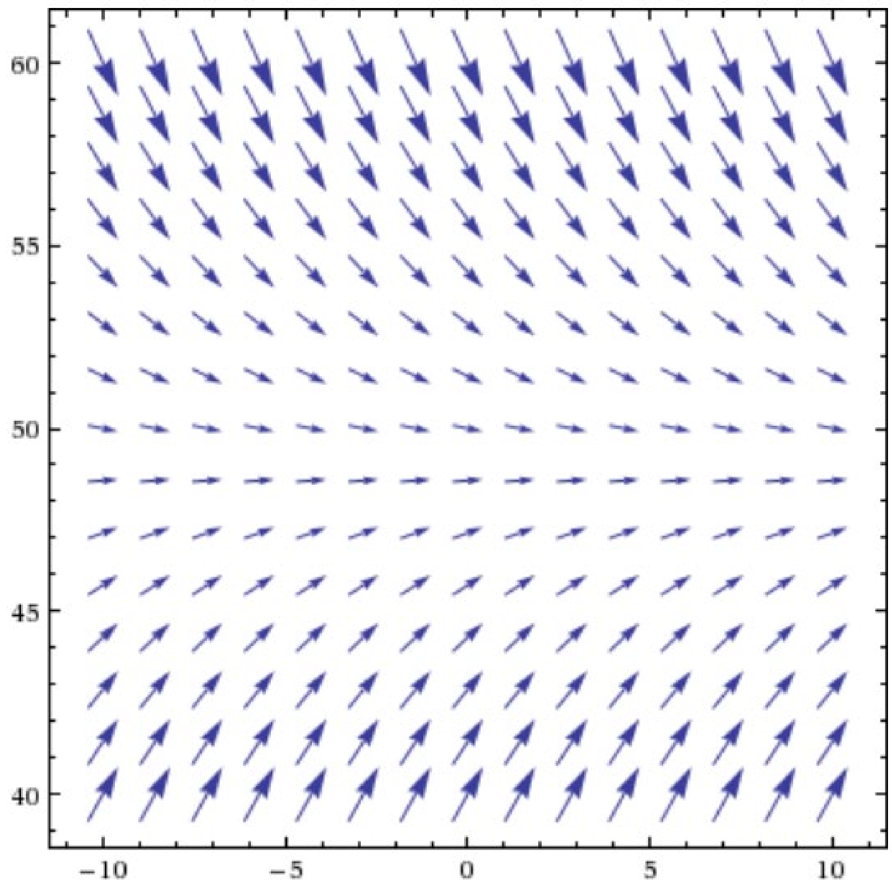

Model 3: Far from earth (no air resistance).

$F_g = -mg\frac{R^2}{(R+x)^2}$ where $R =$ radius of the earth.

If x is small, $\frac{R^2}{(R+x)^2} \sim 1$ and thus $F_g \sim -mg$ when close to earth.

For large x, $mv' = -mg\frac{R^2}{(R+x)^2}$ where R constant.

$\frac{dv}{dt} = -g\frac{R^2}{(R+x)^2}$ with 3 variables: v, t, x

To eliminate one variable: $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$

Note this trick can also be used to simplify some problems.