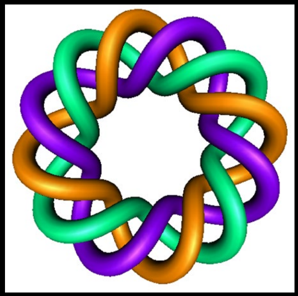
Review of solving first order LINEAR differential equation using an integrating factor



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2.1: First order linear eqn: $y' + p(t)y = g(t)$

Ex 1: $t^2y' + 2ty = sin(t)$
(note, cannot use separation of variables).

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$$(t^2y)' = sin(t) \quad \text{implies} \quad \int (t^2y)'dt = \int sin(t)dt$$

$$(t^2y) = -cos(t) + C$$

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General example: Solve $y' + p(t)y = g(t)$

Let $F(t)$ be an anti-derivative of $p(t)$.

Thus $p(t) = F'(t)$

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