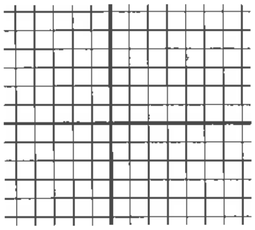
Give that the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

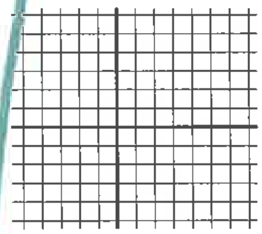
$$\frac{x_2}{x_1} = \frac{3\cancel{e^{-2t}}}{-1\cancel{e^{-2t}}} \Rightarrow x_2 = \frac{3}{-1} x_1$$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the $\Rightarrow c_1 = 0$ & $c_2 = 1$

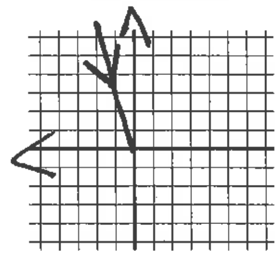
t, x_1-plane $x_1 = -1e^{-2t}$



t, x_2-plane $x_2 = 3e^{-2t}$

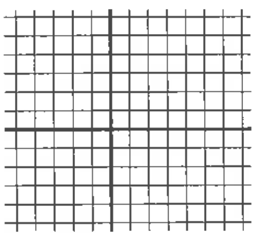


x_1, x_2-plane

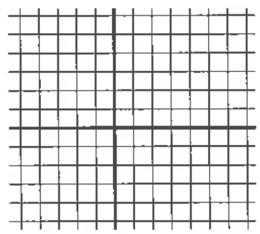


Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

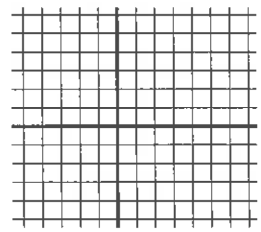
t, x_1-plane



t, x_2-plane



x_1, x_2-plane

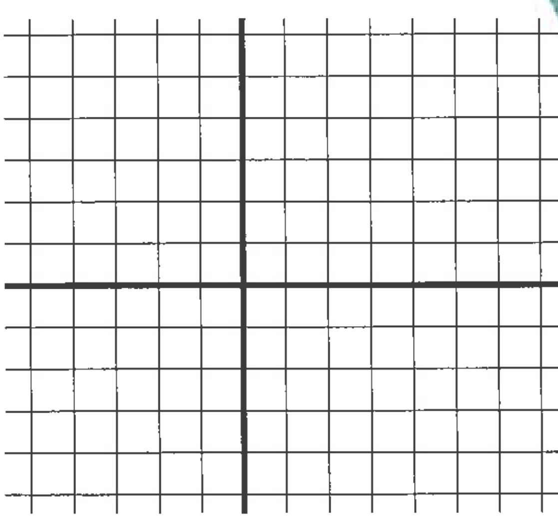


The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$.

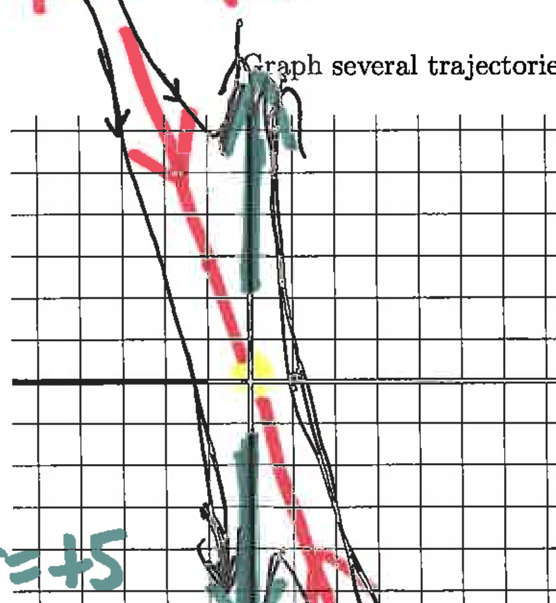
$\frac{dx_2}{dx_1} =$ ______________

$\begin{bmatrix} -1 \\ 3 \end{bmatrix} \to$ slope $3/-1$, $r = -2$

Plot several direction vectors where the slope is 0 and where slope is vertical.



Graph several trajectories.



Saddle

$r = +5$

$\to \begin{bmatrix} 0 \\ 1 \end{bmatrix} \to$ slope $1/0$

$c_1 = c_2 = 0 \Rightarrow \vec{X} = 0$

Give that the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

$\frac{X_2}{X_1} = \frac{1}{0} \Rightarrow X_2 = \frac{1}{0} X_1$

$\frac{X_2}{X_1} = \frac{3}{-1} \Rightarrow X_2 = \frac{3}{-1} X_1$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

t, x_1-plane

t, x_2-plane

x_1, x_2-plane

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

t, x_1-plane

t, x_2-plane

x_1, x_2-plane

The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

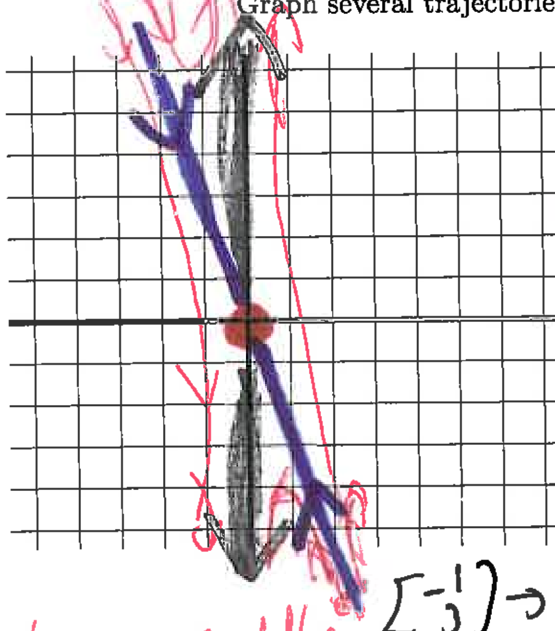
$\begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} \leftarrow$ positive e. value $\Rightarrow$ goes away from

$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow$ slope $\frac{1}{0}$

$\frac{dx_2}{dx_1} =$ ______________

Plot several direction vectors where the slope is 0 and where slope is vertical.

Graph several trajectories.



negative e-value $\Rightarrow$ goes to 0

$\begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2}$

$\begin{bmatrix} -1 \\ 3 \end{bmatrix} \rightarrow$ slope $\frac{3}{-1}$

$t \to +\infty \Rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$ dominate

$t \to -\infty \Rightarrow \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ dominate

Saddle

case

The solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = \overbrace{c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}}^{t \to +\infty} + \overbrace{c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}}^{t \to -\infty}$

Answer the following questions for $A = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = A\mathbf{x}$ is

case: $c_1 \neq 0$ and $c_2 \neq 0$

$t \to +\infty$

For large **positive** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$? $e^{5t} > e^{-2t}$ as $t \to +\infty$

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$,

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

$c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$ + tiny

dominate as $t \to +\infty$

* moves away from the origin. $t \to +\infty$
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ 1/0
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ____________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1 t}$ and $||c_2\mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

$t \to -\infty$

For large **negative** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$? $e^{5t} < e^{-2t}$ as $t \to -\infty$

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

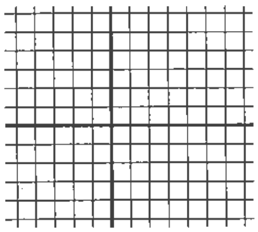
tiny + $c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

* $\boxed{\text{moves away from the origin. } t \to -\infty}$
* ~~moves toward the origin. $t \to +\infty$~~
* approaches the line $y = mx$ with slope $m =$ 3/-1
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ____________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1 t}$ and $||c_2\mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

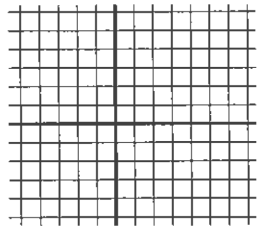
Give that the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

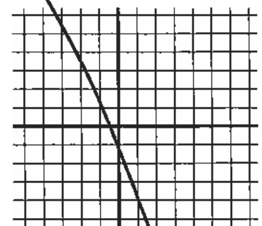
t, x_1-plane



t, x_2-plane

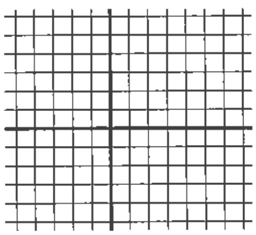


x_1, x_2-plane

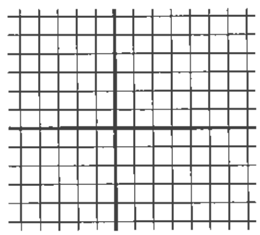


Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

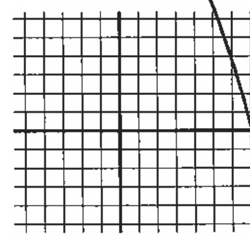
t, x_1-plane



t, x_2-plane



x_1, x_2-plane

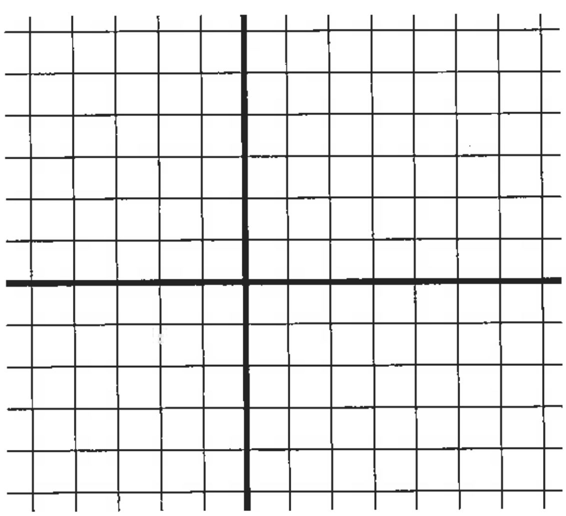


The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

slope $\frac{1}{0}$

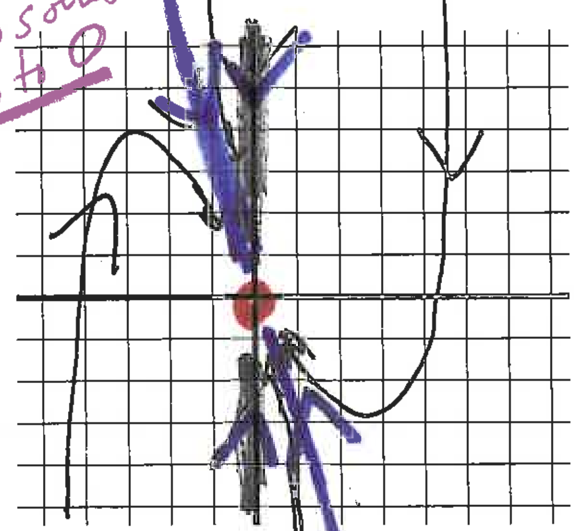
$\frac{dx_2}{dx_1} =$ ______________

Plot several direction vectors where the slope is 0 and where slope is vertical.



negative eigenvalue $\Rightarrow$ soln's go to 0

Graph several trajectories.



both e. vals neg

slope $\frac{3}{-1}$

dominant

the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$. dominates as $t \to +\infty$

as $t \to -\infty$

Answer the following questions for $A = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = A\mathbf{x}$ is

Case for when $c_1 \neq 0$, $c_2 \neq 0$

$t \to +\infty$

For large **positive** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$? $e^{-5t} < e^{-2t}$ as $t \to +\infty$

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$,

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

* moves away from the origin.
* moves toward the origin. $t \to +\infty$
* approaches the line $y = mx$ with slope $m =$ $3/-1$
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ NA. Note this case corresponds to where both $||c_1 \mathbf{v}|| e^{r_1 t}$ and $||c_2 \mathbf{w}|| e^{r_2 t}$ are large, but one is significantly larger than the other.

$\text{tiny} + c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

no displacement

$t \to -\infty$

For large **negative** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$? $e^{-5t} > e^{-2t}$ as $t \to -\infty$

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

* ~~moves away from the origin.~~
* moves toward the origin. $t \to -\infty$
* ~~approaches the line $y = mx$ with slope $m =$~~ NA
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ____. Note this case corresponds to where both $||c_1 \mathbf{v}|| e^{r_1 t}$ and $||c_2 \mathbf{w}|| e^{r_2 t}$ are large, but one is significantly larger than the other.

$c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + \text{big}$

$1/0$

displacement

displa

as $t \to -\infty$

the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ as $t \to +\infty$

Answer the following questions for $A = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = A\mathbf{x}$ is

Case for when $c_1 \neq 0, c_2 \neq 0$

---

as $t \to +\infty$

For large **positive** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$? $e^{-5t} < e^{-2t}$ ← but both small

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$.

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

tiny $+ c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} +$

* moves away from the origin.
* moves toward the origin. as $t \to +\infty$
* approaches the line $y = mx$ with slope $m =$ $3/-1$
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ NA. Note this case corresponds to where both $||c_1 \mathbf{v}||e^{r_1 t}$ and $||c_2 \mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

---

as $t \to -\infty$

For large **negative** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$? $e^{-5t} > e^{-2t}$ ← but both large

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

$\boxed{c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + \text{big}}$

* moves away from the origin. as $t \to -\infty$
* ~~moves toward the origin. as $t \to +\infty$~~
* approaches the line $y = mx$ with slope $m =$ NA
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ $1/0$. Note this case corresponds to where both $||c_1 \mathbf{v}||e^{r_1 t}$ and $||c_2 \mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

will by "parallel" to $\frac{1}{0}$

Give that the solution to $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

t, x_1-plane $\qquad$ t, x_2-plane $\qquad$ x_1, x_2-plane

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

t, x_1-plane $\qquad$ t, x_2-plane $\qquad$ x_1, x_2-plane

The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

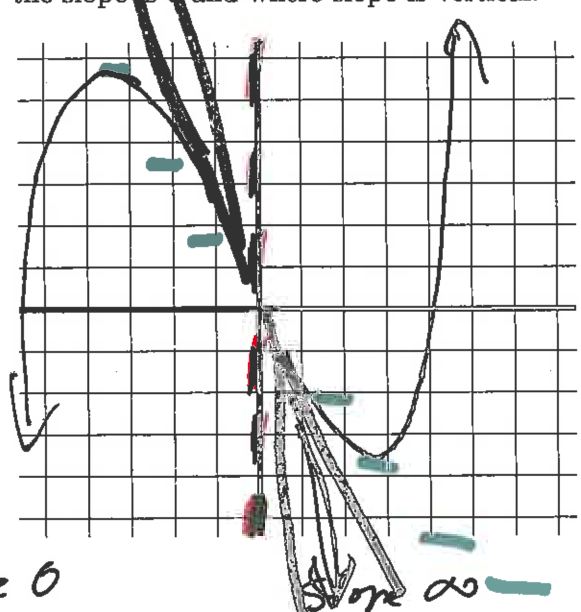
$\frac{dx_2}{dx_1} = \frac{9x_1 + 5x_2}{2x_1}$

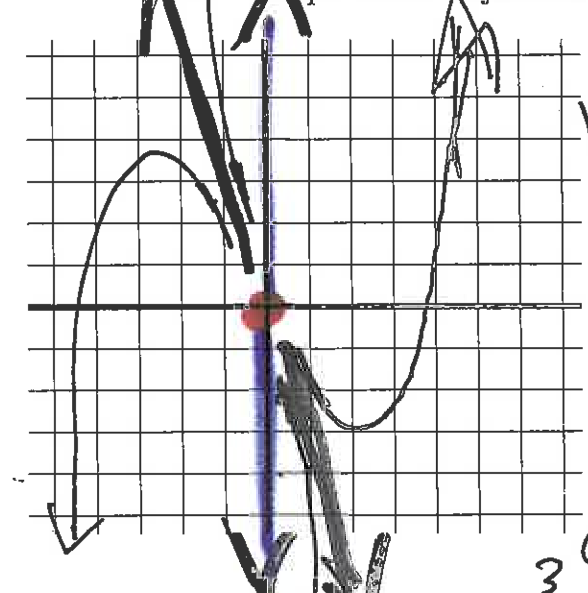
$\frac{1}{0} \leftarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

Plot several direction vectors where the slope is 0 and where slope is vertical.

Graph several trajectories.

both e.value +





slope 0

$x_2 = -\frac{9x_1}{5}$

slope ∞

$x_1 = 0$

positive exp

$\frac{3}{-1} \leftarrow \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$

Give that the solution to $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$

$X_2 = \frac{1}{0} X_1$ $\qquad$ $X_2 = \frac{3}{-1} X_1$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

t, x_1-plane $\qquad$ t, x_2-plane $\qquad$ x_1, x_2-plane

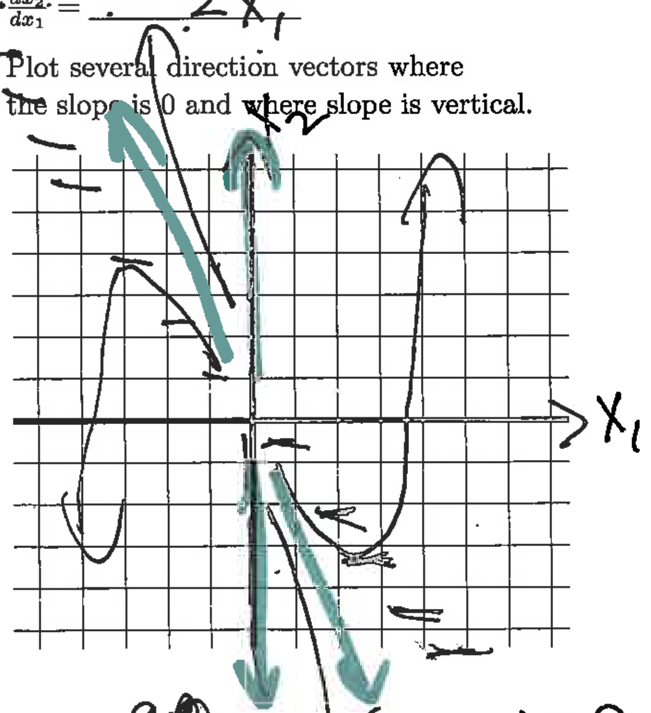
Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

t, x_1-plane $\qquad$ t, x_2-plane $\qquad$ x_1, x_2-plane

The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\frac{dx_2}{dx_1} = \frac{9x_1 + 5x_2}{2x_1}$

Plot several direction vectors where the slope is 0 and where slope is vertical.

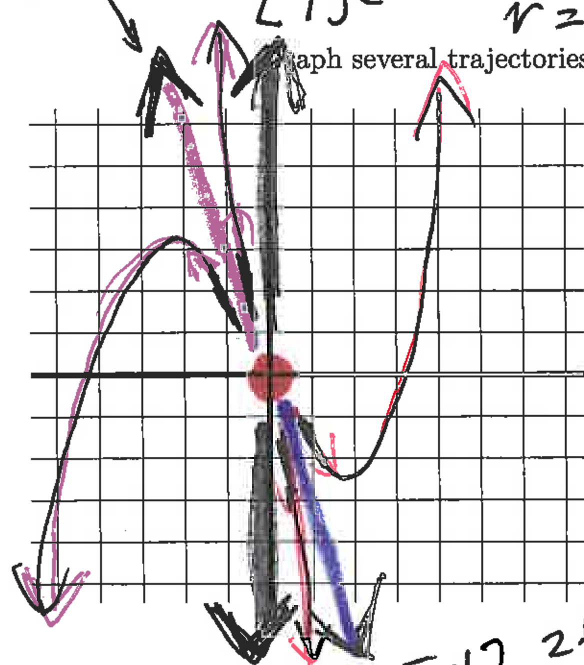


slope 0: $-\frac{9}{5}$ $\qquad$ slope ∞ $X_1 = 0$

e^{2t} dominant as $t \to -\infty$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{+5t}$ $\qquad$ $r = +5$

Graph several trajectories.



$r = +2$

$\begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$

the solution to $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix}\mathbf{x}$ is $\mathbf{x} = \underbrace{c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}}_{\text{domin}}^{t \to +\infty} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$ as $t \to -\infty$ ← dominate

Answer the following questions for $A = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = A\mathbf{x}$ is

case: $c_1 \neq 0$ ~~and~~ and $c_2 \neq 0$

$t \to +\infty$

For large **positive** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$? $e^{5t} > e^{2t}$ as $t \to +\infty$

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$,

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the ~~following behavior (select all~~ that apply):

$\underbrace{c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}}_{\text{huge}} + \text{big}$

* moves away from the origin. $t \to +\infty$
* ~~moves toward the origin.~~
* approaches the line $y = mx$ with slope $m =$ ~~1/0~~ NA
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ 1/0. Note this case corresponds to where both $||c_1 \mathbf{v}||e^{r_1 t}$ and $||c_2 \mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

displaced from $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$ but exhibit similar

displaced but similar behav

$t \to -\infty$

For large **negative** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$? $e^{5t} \ll e^{2t}$ as $t \to -\infty$

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

tiny $+ c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$

* ~~moves away from the origin.~~
* moves toward the origin. $t \to -\infty$
* ~~approaches the line $y = mx$~~ with slope $m =$ 3/-1
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ NA. Note this case corresponds to where both $||c_1 \mathbf{v}||e^{r_1 t}$ and $||c_2 \mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

as $t \to +\infty$

the solution to $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$ $t \to -\infty$

Answer the following questions for $A = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = A\mathbf{x}$ is

case: $c_1 \neq 0$ and $c_2 \neq 0$

$t \to +\infty$

For large **positive** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$? $e^{5t} > e^{2t}$

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$,

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

$\underbrace{c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}}_{\text{huge}} + \underline{\text{big}}$

big

- moves away from the origin. $t \to +\infty$
- moves toward the origin.
- approaches the line $y = mx$ with slope $m =$ ________
- approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ________ 1/0 Note this case corresponds to where both $||c_1 \mathbf{v}||e^{r_1 t}$ and $||c_2 \mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

"parallel" to $y = \frac{1}{0} x$ ← displaced by big

$t \to -\infty$

For large **negative** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$? $e^{5t} < e^{2t}$ for negative value $t \to -\infty$

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

$\text{tiny} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$

- ~~moves away from the origin.~~ $t \to +\infty$
- $\boxed{\text{moves toward the origin. } t \to -\infty}$ 3/-1
- approaches the line $y = mx$ with slope $m =$ ________
- approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ________. Note this case corresponds to where both $||c_1 \mathbf{v}||e^{r_1 t}$ and $||c_2 \mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

$\lim_{t \to -\infty} e^{2t} = 0$

slope field

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 \\ 9x_1 + 5x_2 \end{bmatrix}$$

$$x_1' = \frac{dx_1}{dt} = 2x_1$$

$$x_2' = \frac{dx_2}{dt} = 9x_1 + 5x_2$$

$$\frac{dx_2}{dx_1} = \frac{dx_2}{dt} \cdot \frac{dt}{dx_1} = \frac{dx_2/dt}{dx_1/dt}$$

$$\frac{dx_2}{dx_1} = \frac{9x_1 + 5x_2}{2x_1}$$

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slope 0

$$9x_1 + 5x_2 = 0$$

$$x_2 = -\frac{9x_1}{5}$$

slope ∞

$$2x_1 = 0$$

$$x_1 = 0$$