To solve 1st order linear DE: \[ 1y' + p(t)y = g(t) \]

Create product rule by using integrating factor: \[ u(t) = e^{\int p(t) \, dt} \]

Ex 1: \[ ty' + 9y = \frac{e^{2t}}{t^5} \quad \Rightarrow \quad 1y' + \frac{9}{t}y = \frac{e^{2t}}{t^6} \]

Create product rule by using integrating factor:

\[ u(t) = e^{\int p(t) \, dt} = e^{\int \frac{9}{t} \, dt} = e^{9\ln(t)} = e^{9\ln(t)} = e^{9} \]

Check product rule

\[ (t^9y)' = t^9y' + 9t^8y = t^3e^{2t} \]

\[ (t^9y)' = t^3e^{2t} \quad \Rightarrow \quad \int (t^9y)' = \int t^3e^{2t} \, dt \]

\[ t^9y = \int t^3e^{2t} \, dt = t^3e^{2t} - \int 3t^2e^{2t} \, dt \]

\[ t^9y = t^3e^{2t} - \frac{3t^2e^{2t}}{2} + \frac{6te^{2t}}{8} - \frac{6e^{2t}}{16} + C/t^9 \]

General solution: \[ y = \frac{e^{2t}}{2t^6} - \frac{3e^{2t}}{4t^7} + \frac{3e^{2t}}{4t^8} - \frac{3e^{2t}}{8t^9} + \frac{C}{t^9} \]

Use integration by parts on RHS \( \leq \) right-hand side of equation