To solve 1rst order linear DE: $1y' + \boxed{p(t)}y = g(t)$

Create product rule by using integrating factor: $u(t) = e^{\int \boxed{p(t)} dt}$

Ex 1: $ty' + 9y = \frac{e^{2t}}{t^5}$ $\Rightarrow$ $1y' + \frac{9}{t}y = \frac{e^{2t}}{t^6}$

$p(t) = \frac{9}{t}$

$\int p(t) = \int \frac{9}{t} dt$

$= 9 \ln t$

$e^{9\ln t} = e^{\ln t^9}$

$= t^9$

Create product rule by using integrating factor:

$u(t) = e^{\int p(t)dt} = e^{\int \frac{9}{t} dt} = e^{9ln(t)} = e^{ln(t)^9)} = t^9$

$t^9\left(y' + \frac{9}{t}y\right) = t^9\left(\frac{e^{2t}}{t^6}\right)$

Check product rule

$(t^9 y)' = t^9 y' + 9t^8 y$

product rule

$t^9 y' + 9t^8 y = t^3 e^{2t}$

$(t^9 y)' = t^3 e^{2t}$ $\Rightarrow$ $\int (t^9 y)' = \int t^3 e^{2t} dt$

$(u(t) \cdot y)'$

$t^9 y = \int t^3 e^{2t} dt = t^3 e^{2t} - \int 3t^2 \frac{e^{2t}}{2} dt$

$\frac{t^9 y}{t^9} = \frac{t^3 \frac{e^{2t}}{2}}{t^9} - \frac{3t^2 e^{2t}}{4t^9} + \frac{6te^{2t}}{8t^9} - \frac{6e^{2t}}{16t^9} + C/t^9$

General solution: $y = \frac{e^{2t}}{2t^6} - \frac{3e^{2t}}{4t^7} + \frac{3e^{2t}}{4t^8} - \frac{3e^{2t}}{8t^9} + \frac{C}{t^9}$

Use integration by parts on RHS $\Leftarrow$ right-hand side

$u = t^3$ $\qquad dv = e^{2t}$

$du = 3t^2$ $\qquad v = \frac{e^{2t}}{2}$

$d^2u = 6t$ $\qquad \int v = \frac{e^{2t}}{4}$

$d^3u = 6$ $\qquad \int\int v = \frac{e^{2t}}{8}$

$d^4u = 0$ $\qquad \int\int\int v = \frac{e^{2t}}{16}$