3.5: Non-homogeneous linear differential equation

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$$a(t)y'' + b(t)y' + c(t)y = g(t)$$
 where $g(t) \neq 0$

Step 1: Find homogeneous general solution to a(t)y'' + b(t)y' + c(t)y = 0:

$$y = c_1 \phi_1(t) + c_2 \phi_2(t)$$

Step 2: Find one non-homogeneous solution to a(t)y'' + b(t)y' + c(t)y = g(t):

Step 3: Combine these solutions to create general solution to the non-homogeneous linear DE:

$$y = c_1 \phi_1(t) + c_2 \phi_2(t) + \psi$$

Compare to linear algebra: non-homogenous solution is a "shifted" version of the homogeneous solution (suppose solution space is 2-dimensional):

$$A\mathbf{x} = \mathbf{0}$$
 \Rightarrow $\mathbf{x} = c_1\mathbf{v_1} + c_2\mathbf{v_2}$ \Rightarrow $\mathbf{x} = c_1\mathbf{v_1} + c_2\mathbf{v_2} + \mathbf{w}$ \Rightarrow $\mathbf{x} = c_1\mathbf{v_1} + c_2\mathbf{v_2} + \mathbf{w}$ \Rightarrow $\mathbf{y} = c_1\phi_1(t) + c_2\phi_2(t)$

$$a(t)y'' + b(t)y' + c(t)y = 0 \Rightarrow y = c_1\phi_1(t) + c_2\phi_2(t)$$

$$a(t)y'' + b(t)y' + c(t)y = g(t) \Rightarrow y = c_1\phi_1(t) + c_2\phi_2(t) + \psi$$

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Example 1: Solve $y'' - 4y' + 4y = e^{3t}$
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Step 1: Solve homogeneous version:

$$y'' - 4y' + 4y = 0 \implies r^2 - 4r + 4 = 0 \implies (r - 2)^2 = 0 \implies r = 2, 2$$

Thus general solution to homogeneous DE is $y = c_1 e^{2t} + c_2 t e^{2t}$

Step 2: Find ONE non-homogeneous solution to $y'' - 4y' + 4y = e^{3t}$

3.5 Method of undetermined coefficients – Educated guess: $y = Ae^{3t}$

Plug in to solve for undetermined coefficient A:

$$y = Ae^{3t} \Rightarrow y' = 3Ae^{3t} \Rightarrow y'' = 9Ae^{3t}$$

$$9Ae^{3t} - 4(3Ae^{3t}) + 4Ae^{3t} = e^{3t} \implies 9A - 12A + 4A = 1 \implies A = 1$$

Thus ONE non-homogeneous solution is $y = e^{3t}$

Step 3: Combine these solutions to create general solution to the nonhomogeneous linear DE:

The general solution to nonhomogeneous DE is $y = c_1 e^{2t} + c_2 t e^{2t} + e^{3t}$

Example 2: Solve $y'' - 4y' + 4y = 2\cos(3t)$

Step 1: Solve homogeneous version:

$$y'' - 4y' + 4y = 0 \implies r^2 - 4r + 4 = 0 \implies (r - 2)^2 = 0 \implies r = 2, 2$$

Thus general solution to homogeneous DE is $y = c_1 e^{2t} + c_2 t e^{2t}$

Step 2: Find ONE non-homogeneous solution to $y'' - 4y' + 4y = 2\cos(3t)$

3.5 Method of undetermined coefficients – Educated guess: y = Acos(3t) + Bsin(3t)

Plug in to solve for undetermined coefficient A: $y = A\cos(3t) + B\sin(3t)$ $\Rightarrow y' = -3A\sin(3t) + 3B\cos(3t) \Rightarrow y'' = -9A\cos(3t) - 9B\sin(3t)$

$$y'' = -9Acos(3t) - 9Bsin(3t)$$

 $-4y' = -12Bcos(3t) + 12Asin(3t)$ – Note switched order of B and A term
 $+4y = +4Acos(3t) + 4Bsin(3t)$

$$y'' - 4y' + 4y = (-9A - 12B + 4A)cos(3t) + (-9B + 12A + 4B)sin(3t)$$

$$y'' - 4y' + 4y = (-5A - 12B)cos(3t) + (12A - 5B)sin(3t)$$

$$y'' - 4y' + 4y = 2\cos(3t)$$

$$(-5A - 12B)\cos(3t) + (12A - 5B)\sin(3t) = 2\cos(3t)$$

$$-5A - 12B = 2$$
 and $12A - 5B = 0$

From 2nd equation: 5B = 12A and $B = \frac{12A}{5}$

Plug into first equation: $-5A - 12(\frac{12A}{5}) = 2$

$$-25A - 144A = 10 \implies -169A = 10 \implies A = -\frac{10}{169}$$

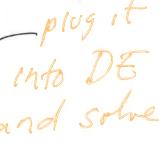
Hence
$$B = (\frac{12}{5})A = B = (\frac{12}{5})(-\frac{10}{169}) = -(\frac{12}{1})(\frac{2}{169}) = -\frac{24}{169}$$

Thus ONE non-homogeneous solution is $y = -\frac{10}{169}cos(3t) - \frac{24}{169}sin(3t)$

Step 3: Combine these solutions to create general solution to the nonhomogeneous linear DE:

The general solution to nonhomogeneous DE is

$$y = c_1 e^{2t} + c_2 t e^{2t} + -\frac{10}{169} \cos(3t) - \frac{24}{169} \sin(3t)$$



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Give the form of the particular (nonhomogenous) solution with undetermined coefficients for

$$y'' + 4y' + 4y = \sin(3t)$$

$$Y(t) = A \cos(3t) + B \sin(3t)$$
(Do NOT solve!)

Give the form of the particular (nonhomogenous) solution with undetermined coefficients for

$$y'' + 4y' + 4y = 4\sin(3t)$$

$$Y(t) = A\cos(3t) + B\sin(3t)$$
 (Do NOT solve!)

Give the form of the particular (nonhomogenous) solution with undetermined coefficients for

$$y'' + 4y' + 4y = \sin(3t) - 8\cos(3t)$$

$$Y(t) = A\cos(3t) + B\sin(3t)$$
 (Do NOT solve!)

Answers:

Give the form of the particular (nonhomogenous) solution with undetermined coefficients for

$$y'' + 4y' + 4y = \sin(3t)$$

$$Y(t) = y = A\cos(3t) + B\sin(3t)$$
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To solve linear DE $ay'' + by' + cy = g_1 + g_2 + g_3$

Step 1: Solve homogeneous version: ay'' + by' + cy = 0 implies $ar^2 + br + c = 0$ implies $y = c_1 \phi_1 + c_2 \phi_2$.

Step 2a: Find one non-homogeneous solution, $y = f_1$, to $ay'' + by' + cy = g_1$ **Step 2b**: Find one non-homogeneous solution, $y = f_2$, to $ay'' + by' + cy = g_2$ **Step 2c**: Find one non-homogeneous solution, $y = f_3$, to $ay'' + by' + cy = g_3$

Step 3: Combine all solutions to create the general solution to the nonhomogeneous DE:

 $y = c_1\phi_1 + c_2\phi_2 + f_1 + f_2 + f_3$ plus it in $\Rightarrow b + g_1 + g_2 + g_3$

Last step: If IVP, plug in initial values to find the constants c_1 and c_2 .

Guess a possible non-homog soln for the following DEs:

Note homogeneous solution to y'' + 2y' + y = 0 is $y = c_1 e^{-t} + c_2 t e^{-t}$ since $r^2 + 2r + 1 = (r+1)(r+1) = 0$

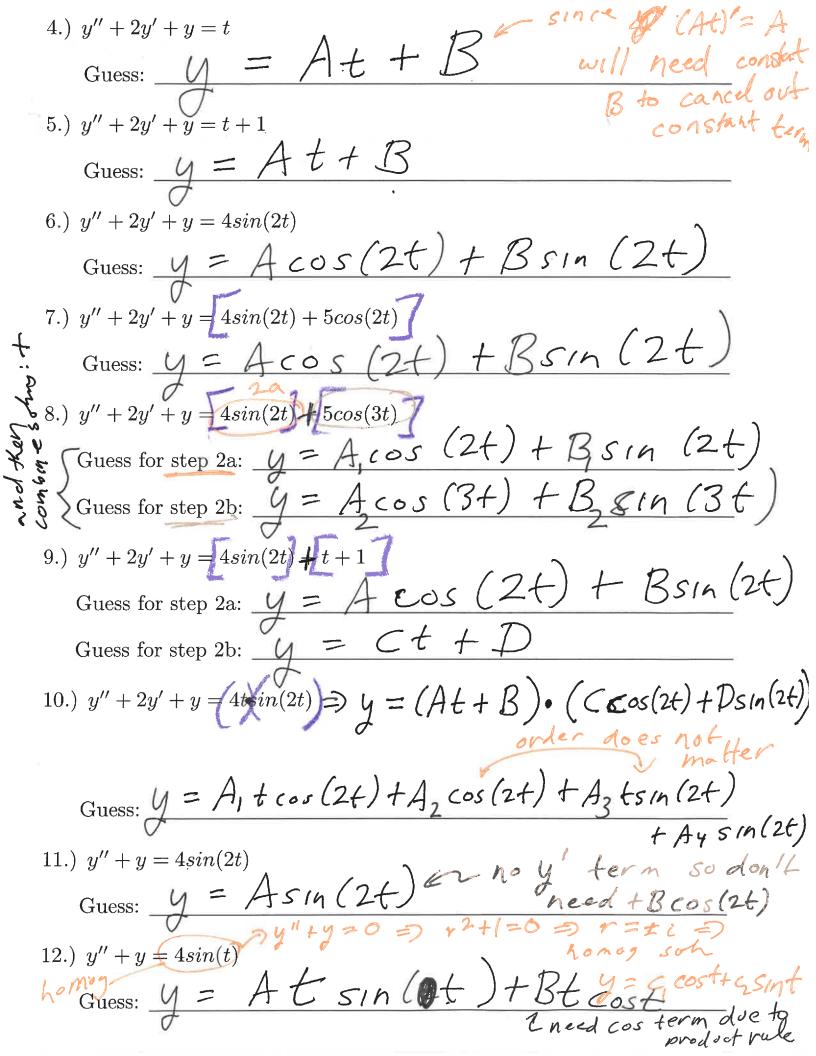
1.) $y'' + 2y' + y = 4e^{2t}$

 $2.) y'' + 2y' + y = 4e^t$

3.) $y'' + 2y' + y = 4e^{-t}$

since homog of sold multiply by tuntil is no longer homog

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$$y'' + y = t^3 \Rightarrow y = At^3 + Bt^2 + Ct + D$$

 $y'' + y' = t^3 = y = [At^3 + Bt^2 + Ct] \neq$

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 $A\mathbf{x} = \mathbf{b} \Rightarrow \mathbf{x} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \mathbf{w}$

$$a(t)y'' + b(t)y' + c(t)y = 0 \Rightarrow y = c_1\phi_1(t) + c_2\phi_2(t)$$

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