Quiz 3 SHOW ALL WORK Name:________________________________

[20] 1.) Solve $y'' + 4y' - 5y = 20 + 12e^t$

Step 1: Solve homogeneous: $y'' + 4y' - 5y = 0$

$r^2 + 4r - 5 = (r + 5)(r - 1) = 0$. Thus $r = -5, 1$. Thus homogeneous solution is $y = c_1e^{-5t} + c_2e^t$

Step 2a: Solve $y'' + 4y' - 5y = 20$

If $y = A$, then $-5A = 20$ and $A = -4$. Thus $y = -4$ is a nonhomogeous solution to $y'' + 4y' - 5y = 20$

Step 2b: Solve $y'' + 4y' - 5y = 12e^t$

$y = e^t$ is a homogeneous solution, so multiply standard guess ($y = Ae^t$) by t.

Let $y = Ate^t$, then $y' = Ae^t + Ate^t$, and $y' = Ae^t + Ae^t + Ate^t = 2Ae^t + Ate^t$

$2Ae^t + Ate^t + 4(Ae^t + Ate^t) - 5Ate^t = 12e^t$

$2Ae^t + Ate^t + 4Ae^t + 4Ate^t - 5Ate^t = 12e^t$

$2Ae^t + 4Ae^t = 12e^t$

$6Ae^t = 12e^t$ and $A = 2$. Thus $y = 2te^t$ is a nonhomogeous solution to $y'' + 4y' - 5y = 12e^t$

Thus the general non-homogeneous solution is $y = c_1e^{-5t} + c_2e^t - 4 + 2te^t$

You were **not** asked to solve an initial value problem, but if you were (for example):

Last step: Solve initial value problem: $y(0) = -4$, $y'(0) = 8$

$y = c_1e^{-5t} + c_2e^t - 4 + 2te^t$: $-4 = c_1 + c_2 - 4$. Thus $0 = c_1 + c_2$ and $c_1 = -c_2$

$y = -5c_1e^{-t} + c_2e^t + 2e^t + 2te^t$: $8 = -5c_1 + c_2 + 2 = 5c_2 + c_2 + 2 = 6c_2 + 2$. Hence $c_2 = 1, c_1 = -1$

Thus the IVP solution is is $y = -e^{-5t} + e^t - 4 + 2te^t$

Answer: $y = c_1e^{-5t} + c_2e^t - 4 + 2te^t$