2.1 Solving 1st order linear diff eqns

Monday, August 31, 2020 2:14 PM

(intro only. See Wednesday for real lesson)

Ex: $t^3 y'(t) + 3t^2 y = 4$ $\boxed{y' = \frac{dy}{dt}}$

check this step

$\checkmark \uparrow$ $\boxed{\text{product rule}}$

$(t^3 y)' = 4$

Get rid of derivative:

$\int (t^3 y)' \, dt = \int 4 \, dt$

Integration by
1. substitution
2. parts
3. partial fractions

$\checkmark$ Easy by FTC

$t^3 y = 4t + C$ $/ t^3$

$\frac{t^3 y}{t^3} = \frac{4t + C}{t^3}$

t is our independent

- integrate with respect to independent

$\int \_\_\_ \, dt$

y is our dependent variable

$y(t)$

$\int y \, dt = ??$

$\int y' \, dt = y + C$

$\int y \, dt = y + c$

general soln $\boxed{y = 4t^{-2} + Ct^{-3}}$

HW 2

2.2 : (1) $y' = \frac{x^2}{y}$

||

1.2

Quiz 1 ~~will~~ open~~this evening~~ed yesterday. It is a real quiz that must be finished within 30 minutes after you open the quiz (but I think I have given you plenty of time for quiz 1). **It is due this week on Thursday (11:59pm).** tommorrow The quiz focuses on questions relating to the pre-recorded videos from last week (see week 1 at https://homepage.divms.uiowa.edu/~idarcy/COURSES/100/SLIDES/videos.html). Note for this quiz you are allowed to use any source you find, except for other people. Thus you might want the slides open from week 1 pre-recorded videos before starting quiz 1. I'll mention quiz 1 in class tomorrow as well.

The Math Lab is open for

- in-person tutoring during **day**time hours and
- online tutoring via Zoom during the **day and evening.**

2.1: Integrating factor

How to create product rule for 1rst order LINEAR DE

$$\boxed{1 y' + p(t)\, y = g(t)}$$

EX: $\frac{t^3}{t^3} y' + \frac{3t^2}{t^3} y = \frac{4}{t^3}$ ← Linear but not seperable so must use 2.1 technique

If you don't see product rul $\div t^3$

notice product rule

$$1 y' + \left(\frac{3}{t}\right) y = \frac{4}{t^3} \qquad (t^3 y)' = 4$$

↑ $p(t)$

Integrator factor

$$u(t) = e^{\int p(t)\,dt}$$

$p(t)$ = coef of y in DE $1y' + \underbrace{p(t)}\cdot y = g(t)$

$p(t) = \frac{3}{t}$

$$\boxed{u(t) = e^{\int p(t)\,dt}} = e^{\int \frac{3}{t}\,dt} = e^{3\int \frac{1}{t}\,dt}$$

... one

$$\boxed{u(t) = e^{\int p}}$$

$$= e^{3\ln|t| \cancel{+C}}$$ ← only need one $u(t)$

Know this formula

$$= e^{\ln|t|^3} = |t|^3$$

$|t|^3 + C$ Don't need most general form but don't need most general form to create product rule

Let $u(t) = t^3$

$$\left[y' + \frac{3}{t}y = \frac{4}{t^3} \right] t^3$$

$$y't^3 + 3t^2y = 4$$

product rule

$$\int (yt^3)' \, dt = \int 4 \, dt$$

$u(t)$ = integrating factor

$$yt^3 = 4t + C$$

$$\boxed{y = \frac{4}{t^2} + \frac{c}{t^3}}$$

integrating factor

general soln to 1rst order DE

$\exists c$

In general

$$y' + \underbrace{p(t)} y = g(t)$$

Let $\boxed{u(t) = e^{\int p(t)\,dt}} = e^{F(t)}$

Let $F(t)$ = an antiderivative of $p(t)$

$$\Rightarrow \underline{F'(t) = p(t)}$$

$$\left[\, y' + p(t)\, y = g(t) \,\right] e^{F(t)}$$

$$e^{F(t)} y' + p(t) e^{F(t)} y = g(t) e^{F(t)}$$

$$\underbrace{e^{F(t)}}\underbrace{y'} + \underbrace{F'(t)e^{F(t)}y} = g(t)e^{F(t)}$$

✓ product

So this proves we can always create product rule for 1rst order LINEAR DE

$$\int \left(y \cdot \underbrace{e^{F(t)}}\right)' dt = \int g(t)e^{F(t)}\,dt$$

$u(t) = e^{F(t)}$

$$y \cdot e^{F(t)} = \int g(t)e^{F(t)}\,dt$$

$$y = e^{-F(t)} \int g(t)e^{F(t)}\,dt$$

we have also proven that we can always solve for y and thus give explicit general sol'n

Don't forget $+C$ after integratin RHS

IVP: Ex 2: $y' + 3t^2 y = t^2$

$$y(0) = 2$$

First solve DE $y' + 3t^2 y = t^2$

2nd: plug in initial value $y(0) = 2$ into general soln to find C

$$1\, y' + \underbrace{3t^2}\, y = t^2$$

$$p(t) = 3t^2 \Rightarrow \int 3t^2 = t^3$$

$$u(t) = e^{\int 3t^2\, dt} = e^{t^3}$$

$$\left[y' + 3t^2 y = t^2 \right] e^{t^3}$$

$$y' e^{t^3} + \underline{3t^2 e^{t^3}}\, y = t^2 e^{t^3}$$

$\uparrow$ ✓

$$\int (y e^{t^3})'\, dt = \int t^2 e^{t^3}\, dt$$

$$ye^{t^3} = \frac{1}{3}\int \underbrace{3t^2 e^{t^3}\,dt}$$

Let $u = t^3$, $du = \underbrace{3t^2 dt}$

$$\frac{du}{3} = t^2 dt$$

$$ye^{t^3} = \int (e^u)\frac{du}{3}$$

$$= \frac{1}{3}e^u + C$$

$$\frac{ye^{t^3}}{e^{t^3}} = \frac{\frac{1}{3}e^{t^3}}{e^{t^3}} + \frac{C}{e^{t^3}}$$

$$\boxed{\text{General soln} \quad y = \frac{1}{3} + Ce^{-t^3}}$$

IVP: $y(0) = 2$

continue next time

2.1

(1) $y' + 3y = te^{-2t}$

Breakout room