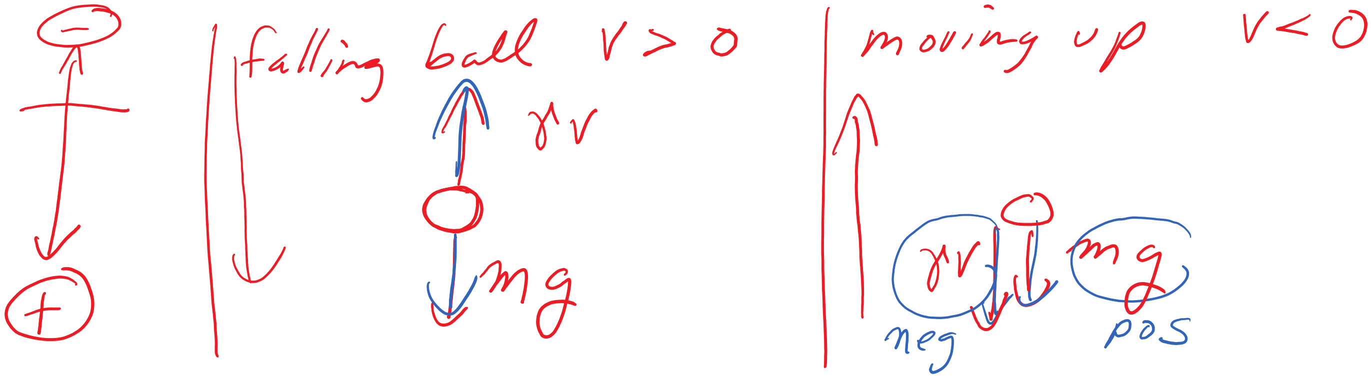
1.1 Ball example



$$\overset{ma}{mv'} = mg - \gamma v$$

Direction field

ex: $m = 10$, $\gamma = 2$

$$\Rightarrow \boxed{v' = 9.8 - \frac{v}{5}}$$

Are there any equilibrium soln?

Equilibrium soln $\equiv$ A constant soln

$$\boxed{v = c} \iff v' = 0$$

To find equil solns, set $v' = 0$

$v' = 9.8 - \frac{v}{5}$. If $v' = 0 \Rightarrow 0 = 9.8 - \frac{v}{5}$

$\Rightarrow \frac{v}{5} = 9.8 \Rightarrow v = (9.8)(5) = \underline{49}$

$$\Rightarrow \frac{v}{5} = 9.8 \Rightarrow v = (9.8)(5) = 49$$

Equil soln is $v = 49$

Check by plugging in $v' = 9.8 - \frac{v}{5}$

$\boxed{v = 49} \Rightarrow \boxed{v' = 0}$ $\qquad 0 \overset{?}{=} 9.8 - \frac{49}{5}$ ✓

$v' = 9.8 - \frac{v}{5}$

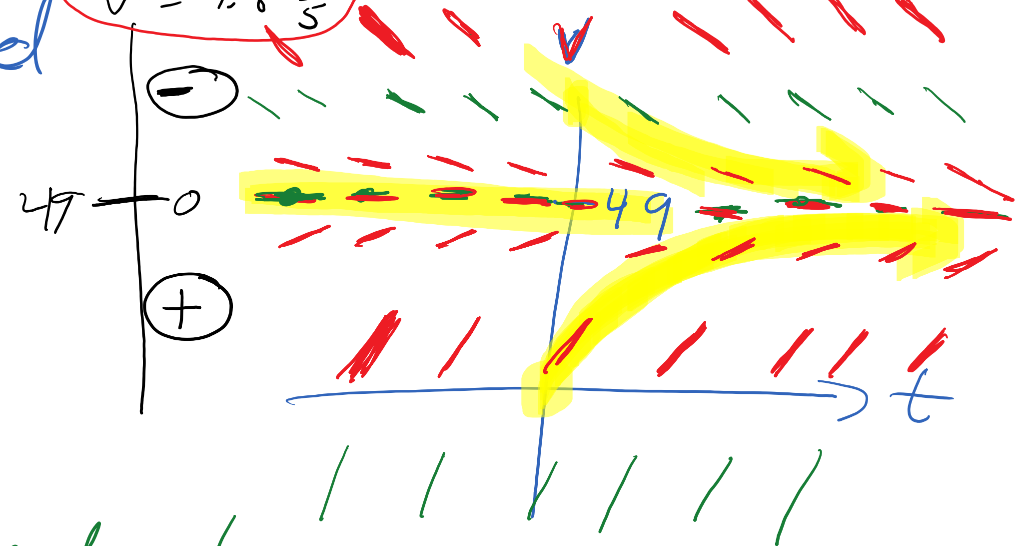
Direction field

small portions of tangent lines

= slope field

$$\boxed{v' = 9.8 - \frac{v}{5}}$$

does not depend on t



1.2 = 2.2 : Solving DE via separation of variables

$$dt\left(\frac{dv}{dt}\right) = \left(9.8 - \frac{v}{5}\right) dt$$

$$\cancel{\frac{1}{v}}\,(dv) = \left(\left(9.8 - \frac{v}{5}\right) dt\right) \cancel{\frac{1}{v}}$$

$$5(dv) = \left[\left(9.8 - \frac{v}{5}\right) dt\right] 5$$

$$\frac{5\,dv}{(49-v)} = \frac{\cancel{(49-v)}\cdot dt}{\cancel{(49-v)}}$$

$$\int \left(\frac{5}{49-v}\right)\cdot(dv) = \int 1 \cdot dt$$

height width

Separation variables means turn into $\boxed{\text{calc 1 problem}}$ so dv & dt are never in denominator

$$\boxed{5\int \frac{dv}{49-v} = \int dt}$$

Let $u = 49-v$ $\quad du = -dv$

can use u-substition
or do it in your head & check

$$-5 \ln|49-v| = t + C$$

$$\frac{-5 \ln|49-v|}{-5} = \frac{t + C}{-5}$$

Solve for v:

$$\ln|49-v| = \frac{-t}{5} + C$$

sloppy is good!

$\frac{C}{-5}$ is a constant

$e \qquad e \qquad \uparrow$

$e \qquad e^{\frac{}{5}} \quad C \quad \to$ constant

This C swallowed $-\frac{1}{5}$

$$|49 - v| = e^{-t/5 + C}$$

$$= e^{-t/5}(e^{C})$$

$$|49 - v| = Ce^{-t/5}$$

$e^{C} > 0$

but C can be any real #

$$49 - v = (\pm C)e^{-t/5}$$

$$49 - 49 - v = Ce^{-t/5} - 49$$

$$-(-v) = -(Ce^{-t/5} - 49)$$

$$\boxed{v = Ce^{-t/5} + 49}$$

general soln

velocity vs time

In our class don't need to worry about loosing or gaining solns

BUT in real world, you do

initial value problem (IVP)

DE: $v' = 9.8 - \frac{v}{5}$

initial value: $v(t_0) = v_0$

Example: $v(0) = 0$

For example, if drop ball, $v(0) = 0$

Solve IVP

① Solve DE for general sol'n

$v = 49 - Ce^{-t/5}$ $\Leftrightarrow$ $\boxed{v = Ce^{-t/5} + 49}$

② Plug in initial value to find C

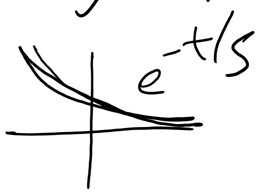
$\underset{t=0}{v(0)} = 0 \Rightarrow t = 0,\ v = 0$

$$0 = Ce^{0} + 49$$

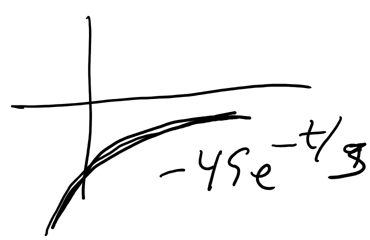
$$0 = C + 49 \Rightarrow C = -49$$

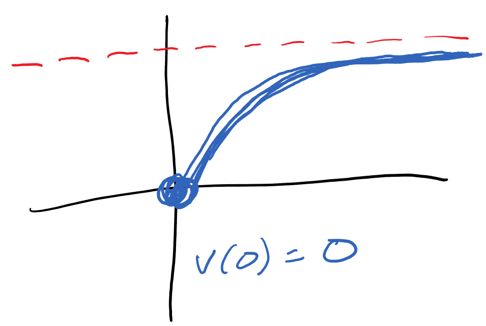
$$\boxed{\text{IVP soln: } v = -49e^{-t/5} + 49}$$

graph the soln:



$\rightarrow$





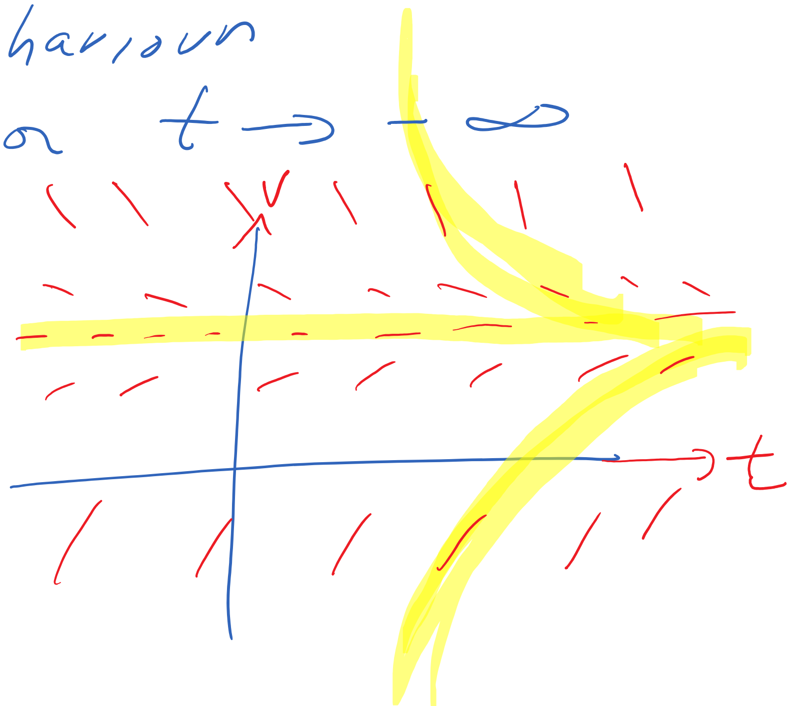
$e^{-t/5} \to 0$ as $t \to \infty$

$v \to 0 + 49 = 49$ as $t \to \infty$

Direction field

Long-term behaviour

$t \to +\infty$ or $t \to -\infty$



as $t \to +\infty$

$v \to 49$

as $t \to -\infty$

If $v_0 > 49$

$v \to +\infty$

If $v_0 < 49$, $v \to -\infty$