Quiz 2 Section 93 Sept 27, 2019

1.) Circle the correct answer:

[0.25] 1i.) If $y(0) = -2$ and $y^2 = g(t)$, then $y(t) = \sqrt{g(t)}$

A) True B) False

[0.25] 1ii.) If $y(0) = -2$ and $y^2 = g(t)$, then $y(t) = -\sqrt{g(t)}$

A) True B) False

[0.5] 1iii.) If $y(-1) = 3$ and $y^2 = g(t)$, then $y(t) = \sqrt{g(t)}$

A) True B) False

[0.5] 1iv.) When taking the derivative with respect to t, $(y^2)' = 2y$

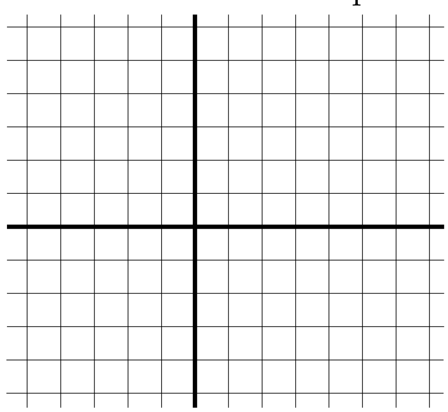
A) True B) False

[0.5] 1v.) When taking the derivative with respect to t, $(y^2)' = 2yy'$

A) True B) False

---

[3] 2a.) Sketch the direction field for the autonomous equation $y' = (y-3)^5(y+2)^2$



[1] 2b.) On the graph above, sketch the solution with initial value $y(0) = 0$

[2] 2c.) Find the equilibrium solutions, and classify them as asymptotically stable or unstable or semi-stable.

Equilibrium solution: __________. Stability of this equilibrium solution ______________.

Equilibrium solution: __________. Stability of this equilibrium solution ______________.

---

[3] 3.) The solution to the initial value problem $y' = \frac{\sqrt{1+y^2}}{x}$, $y(1) = 0$ is $y = \frac{x^2-1}{2x}$
State the largest interval on which the solution is defined.

4.) State the initial value problem describing the following. **Do NOT solve.**
Suppose salty water enters and leaves a tank at a rate of 2 liters/minute. Suppose also that the salt concentration of the water entering the tank is 3 g/liters. If the tank contains 4 liters of water and initially contains 5g of salt, find a formula for the amount of salt in the tank after t minutes.

[4] Differential equation: ____

[1] Initial Value: ____

[4] 5.) Solve the following first order linear differential equation: $y' + \frac{5y}{x} = 1$

General Solution: ____

Quiz 2 Section 93 Sept 27, 2019

1.) Circle the correct answer:

[0.25] 1i.) If $y(0) = -2$ and $y^2 = g(t)$, then $y(t) = \sqrt{g(t)}$

A) True B) False

[0.25] 1ii.) If $y(0) = -2$ and $y^2 = g(t)$, then $y(t) = -\sqrt{g(t)}$

A) True B) False

[0.5] 1iii.) If $y(-1) = 3$ and $y^2 = g(t)$, then $y(t) = \sqrt{g(t)}$

A) True B) False

[0.5] 1iv.) When taking the derivative with respect to t, $(y^2)' = 2y$

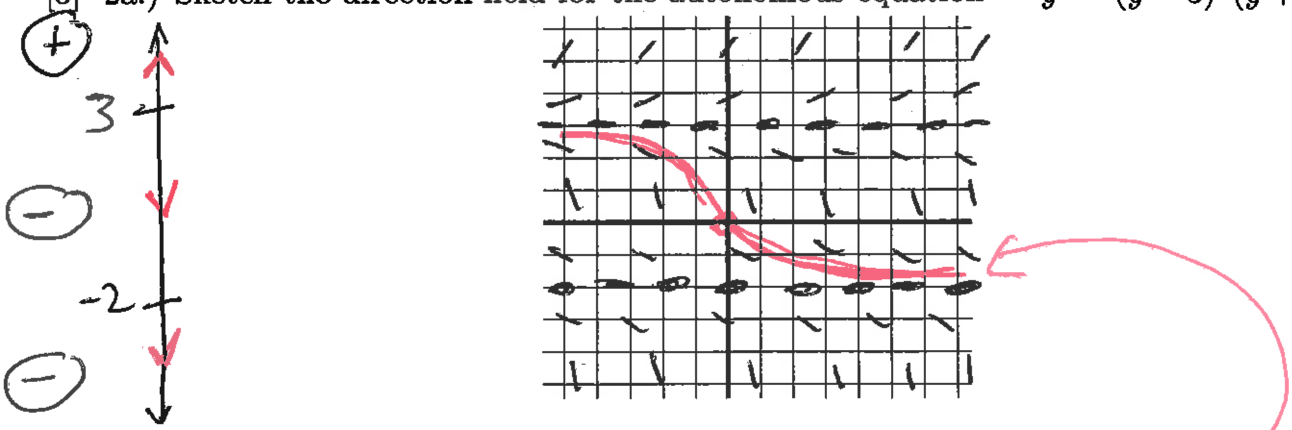
A) True B) False

[0.5] 1v.) When taking the derivative with respect to t, $(y^2)' = 2yy'$

A) True B) False

---

[3] 2a.) Sketch the direction field for the autonomous equation $y' = (y-3)^5(y+2)^2$



[1] 2b.) On the graph above, sketch the solution with initial value $y(0) = 0$

[2] 2c.) Find the equilibrium solutions, and classify them as asymptotically stable or unstable or semi-stable.

Equilibrium solution: $y = 3$. Stability of this equilibrium solution unstable.

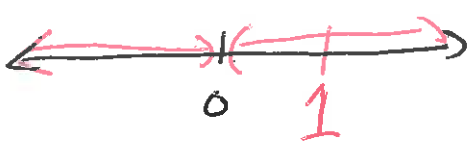
Equilibrium solution: $y = -2$. Stability of this equilibrium solution semi-stable.

---

[3] 3.) The solution to the initial value problem $y' = \frac{\sqrt{1+y^2}}{x}$, $y(1) = 0$ is $y = \frac{x^2-1}{2x}$. State the largest interval on which the solution is defined.

$x \neq 0$

$t_0 = 1 \in (0, \infty)$



Answer: $(0, \infty)$

4.) State the initial value problem describing the following. **Do NOT solve.**
Suppose salty water enters and leaves a tank at a rate of 2 liters/minute. Suppose also that the salt concentration of the water entering the tank is 3 g/liters. If the tank contains 4 liters of water and initially contains 5g of salt, find a formula for the amount of salt in the tank after t minutes.

Answer: Let $Q(t)$ = amount of salt measured in grams in the tank at time t (measured in minutes).

At time $t = 0$, $Q(0) = 5$

Looking for $\frac{dQ}{dt}$ in grams/minute.

Rate in: $(\frac{2\ liters}{minute})(\frac{3\ grams}{liter}) = \frac{6\ grams}{minute}$

Note at time t, there are $Q(t)$ grams of salt in 4 liters. Thus concentration of salt in water flowing out at time t is $\frac{Q(t)\ grams}{4\ liters}$

Rate out: $(\frac{2\ liters}{minute})(\frac{Q(t)\ grams}{4\ liters}) = \frac{Q\ grams}{2\ minute}$

$\frac{dQ}{dt}$ = Rate in - Rate out $= 6 - \frac{Q}{2}$

[4] Differential equation: $Q' = 6 - \frac{Q}{2}$

[1] Initial Value: $Q(0) = 5$

[4] 5.) Solve the following first order linear differential equation: $y' + \frac{5y}{x} = 1$

$1y' + \frac{5y}{x} = 1$. Let $u = e^{\int \frac{5dx}{x}} = e^{5ln|x|} = e^{ln|x|^5} = |x|^5$

Let $u(x) = x^5$

$x^5y' + 5x^4y = x^5$

$(x^5y)' = x^5$ CHECK PRODUCT RULE to make sure this step is correct.

$\int (x^5y)'dx = \int x^5dx$

$x^5y = \frac{x^6}{6} + C$

$y = \frac{x}{6} + Cx^{-5}$

Check: $y' = \frac{1}{6} - 5Cx^{-6}$ and thus $y' + \frac{5y}{x} = \frac{1}{6} - 5Cx^{-6} + \frac{5}{6} + 5Cx^{-6} = 1$

General Solution: $y = \frac{x}{6} + Cx^{-5}$