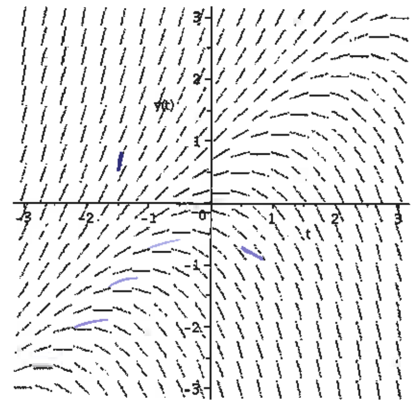
$y' = f(t, y)$

4.) Circle the general solution to the differential equation whose direction field is given below:



A) $y = t + C$ **B)** $y = t^2 + C$

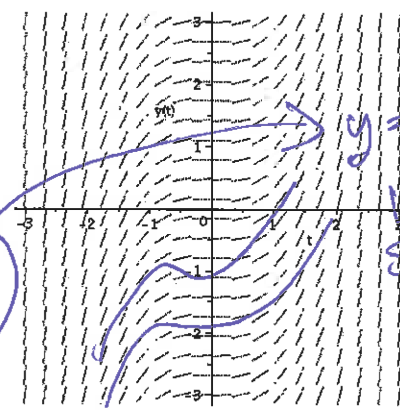
C) $y = e^t + C$ **D)** $y = Ce^t + t + 1$

E) $y = Ce^t$ **F)** $y = e^t + t + C$

G) $y = ln(t) + C$ **H)** $y = C$

I) $y = sin(t) + C$ **J)** $y = cos(t) + C$

5.) Which of the following could be the general solution to the differential equation whose direction field is given below:



A) $y = t + C$ **B)** $y = t^2 + C$

C) $y = e^t + C$ **D)** $y = \frac{(t-1)^3}{3} + C$

E) $y = Ce^t$ **F)** $y = \frac{t^3}{3} + C$

G) $y = ln(t) + C$ **H)** $y = C$

I) $y = \frac{Ct^3}{3}$ **J)** $y = \frac{C(t-1)^3}{3}$

$y' = f(t)$

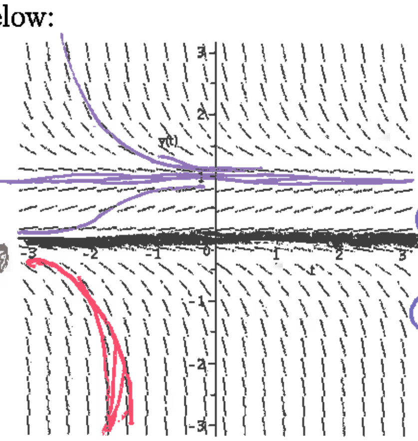
$y' = t^2$

Calc 1 problem

$y = f(t) + C$

vertical shifts

6.) Circle the differential equation whose direction field is given below:



~~**A)** $y' = t^2$~~ ~~**B)** $y' = y + 3$~~

~~**C)** $y' = e^t$~~ ~~**D)** $y' = t + 1$~~

~~**E)** $y' = t - y$~~ ~~**F)** $y' = y - t$~~

~~**G)** $y' = (1 + y)(1 - y)$~~ **H)** $y' = y(1 + y)$

~~**I)** $y' = t(1 - t)$~~ $\boxed{\textbf{J)}\ y' = y(1 - y)}$

$-$

$\leftarrow$ equ

$+$

$-$

$y = 1$ stable

$y = 0$ unstable

$y' = f(y)$

2.5 Autonomous DE

2.8 initial value will always be $y(0)=0$ } only for §2.8

2.8: Approximating solution using

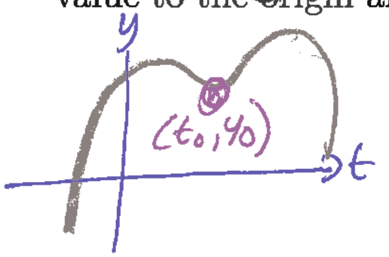
Method of Successive Approximation

(also called Picard's iteration method).

translate back

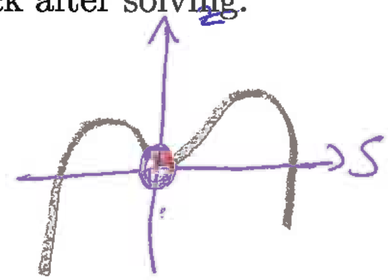
IVP: $y' = f(t,y)$, $y(t_0) = y_0$.

Note: Can always translate IVP to move initial value to the origin and translate back after solving:



translate to origin

Let $s = t - t_0$
$z = y - y_0$



Hence for simplicity **in section 2.8**, we will assume initial value is at the origin: $y' = f(t,y)$, $y(0) = 0$.

Thm 2.4.2: Suppose the functions $z = f(t,y)$ and $z = \frac{\partial f}{\partial y}(t,y)$ are continuous on $(a,b) \times (c,d)$ and the point $(t_0, y_0) \in (a,b) \times (c,d)$,

then there exists an interval $(t_0 - h, t_0 + h) \subset (a,b)$ such that there exists a unique function $y = \phi(t)$ defined on $(t_0 - h, t_0 + h)$ that satisfies the following initial value problem:

$$\boxed{y' = f(t,y), \quad y(t_0) = y_0.}$$

Thm 2.8.1 is translated to origin version of Thm 2.4.2:

Thm 2.8.1: Suppose the functions $z = f(t,y)$ and $z = \frac{\partial f}{\partial y}(t,y)$ are continuous for all t in $(-a,a) \times (-c,c)$,
then there exists an interval $(-h,h) \subset (-a,a)$ such that there exists a unique function $y = \phi(t)$ defined on $(-h,h)$ that satisfies the following initial value problem:

$$\boxed{y' = f(t,y), \quad y(0) = 0.}$$

Proof outline (note this is a constructive proof and thus the proof is very useful).

Given: $y' = f(t,y)$, $y(0) = 0$ Eqn (*)
f, $\partial f / \partial y$ continuous $\forall (t,y) \in (-a,a) \times (-b,b)$.

Then $y = \phi(t)$ is a solution to (*) iff

$\phi'(t) = f(t, \phi(t))$, $\phi(0) = 0$ iff

$\int_0^t \phi'(s)ds = \int_0^t f(s,\phi(s))ds$, $\phi(0) = 0$ iff

$\phi(t) = \phi(t) - \phi(0) = \int_0^t f(s, \phi(s))ds$

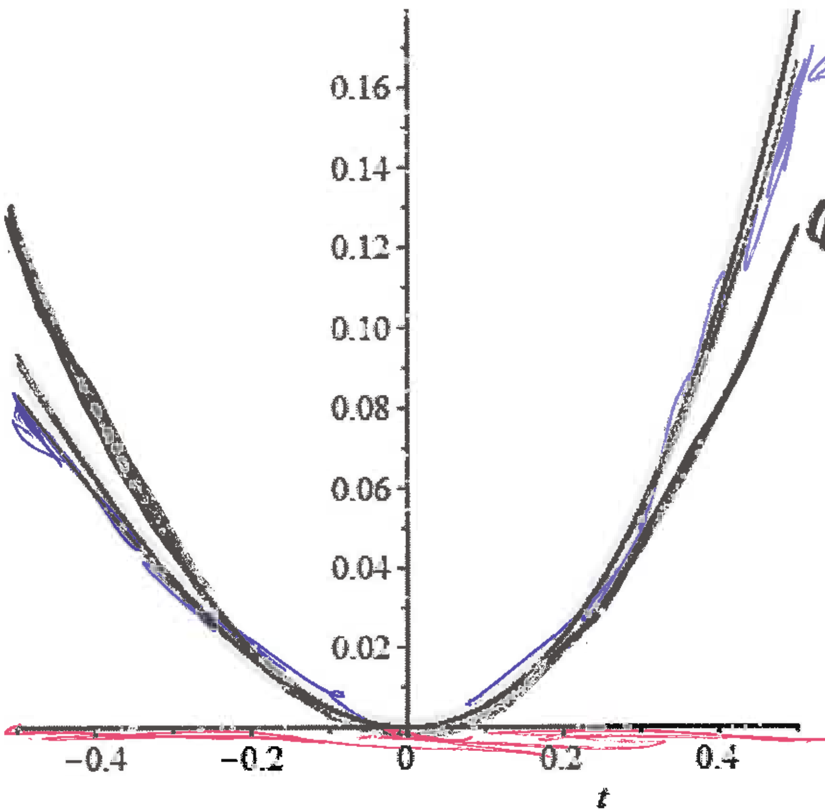
Thus $y = \phi(t)$ is a solution to (*)
iff $\phi(t) = \int_0^t f(s, \phi(s))ds$

Sequence of functions that approaches sol'n

2.8: Approximating soln to IVP using seq of fns.

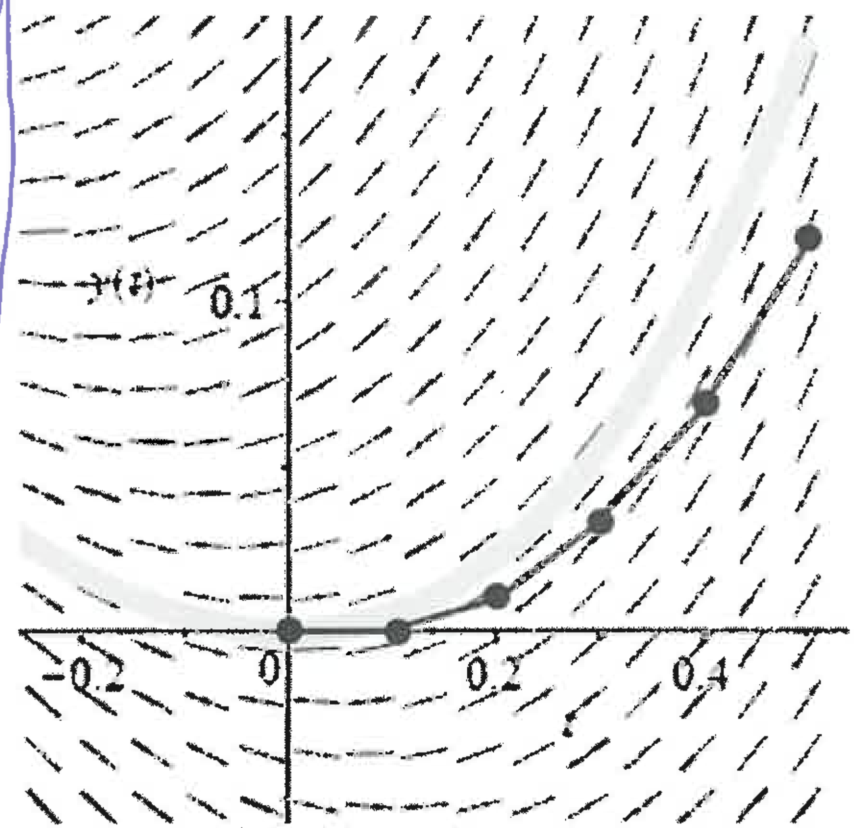
$\phi_0(t) = 0$, $\phi_1(t) = \frac{t^2}{2}$, $\phi_2(t) = \frac{t^2}{2} + \frac{t^3}{3}$,

$\phi_3(t) = \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{6}$, $\phi_4(t) = \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{6} + \frac{t^5}{15}$



2.7: Approximating soln to IVP using multiple tangent lines.

$$y(t) = \begin{cases} 0 & 0 \le t \le 0.1 \\ 0.1t - 0.01 & 0.1 \le t \le 0.2 \\ 0.22t - 0.034 & 0.2 \le t \le 0.3 \\ 0.364t - 0.0772 & 0.3 \le t \le 0.4 \\ 0.5328t - 0.14672 & 0.4 \le t \le 0.5 \end{cases}$$



approx soln

Sequence

$y' = f(t,y)$

Construct ϕ using method of successive approximation – also called Picard's iteration method.

Let $\phi_0(t) = 0$ (or the function of your choice)

Let $\phi_1(t) = \int_0^t f(s, \phi_0(s))ds$

Let $\phi_2(t) = \int_0^t f(s, \phi_1(s))ds$

⋮

Let $\phi_{n+1}(t) = \int_0^t f(s, \phi_n(s))ds$

Let $\boxed{\phi(t) = \lim_{n\to\infty} \phi_n(t)}$

Claim ϕ is the unique soln to IVP

To finish the proof, need to answer the following questions (see book or more advanced class):

1.) Does $\phi_n(t)$ exist for all n? ✓

2.) Does sequence ϕ_n converge?

use Ratio test for HW

3.) Is $\phi(t) = \lim_{n\to\infty} \phi_n(t)$ a solution to (*).

4.) Is the solution unique.

$\phi_{n+1}(t) = \int_0^t f(s, \phi_n(s))ds$

Example: $y' = t + 2y$. That is $f(t, y) = t + 2y$

Let $\phi_0(t) = 0$

Let $\phi_1(t) = \int_0^t f(s, 0)ds = \int_0^t (s + 2(0))ds$

$= \int_0^t s\,ds = \frac{s^2}{2}\Big|_0^t = \frac{t^2}{2}$

Let $\phi_2(t) = \int_0^t f(s, \phi_1(s))ds = \int_0^t f(s, \frac{s^2}{2})ds$

$= \int_0^t (s + 2(\frac{s^2}{2}))ds = \frac{t^2}{2} + \frac{t^3}{3}$

Let $\phi_3(t) = \int_0^t f(s, \phi_2(s))ds = \int_0^t f(s, \frac{s^2}{2} + \frac{s^3}{3})ds$

$= \int_0^t (s + 2(\frac{s^2}{2} + \frac{s^3}{3}))ds = \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{6}$

Let $\phi_4(t) = \int_0^t f(s, \phi_3(s))ds$

$= \int_0^t f(s, \frac{s^2}{2} + \frac{s^3}{3} + \frac{s^4}{6})ds$

$= \int_0^t (s + 2(\frac{s^2}{2} + \frac{s^3}{3} + \frac{s^4}{6}))ds$

$= \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{6} + \frac{t^5}{15}$

⋮

$\phi_n(t) =$ for arbitrary n

Take limit as $n \to \infty$