Quiz 4   SHOW ALL WORK
Nov 9, 2018

[15] 1.) Solve $ty' + 4y = t$

$1y' + \frac{4}{t}y = 1$

$u(t) = e^{\int \frac{4}{t} dt} = e^{4 \ln|t|} = e^{\ln(|t|^4)} = t^4$.

Let $u(t) = t^4$

$t^4 y' + 4t^3 y = t^4$

$(t^4 y)' = t^4$ **Check this step:** $(t^4 y)' = t^4 y' + 4t^3 y$

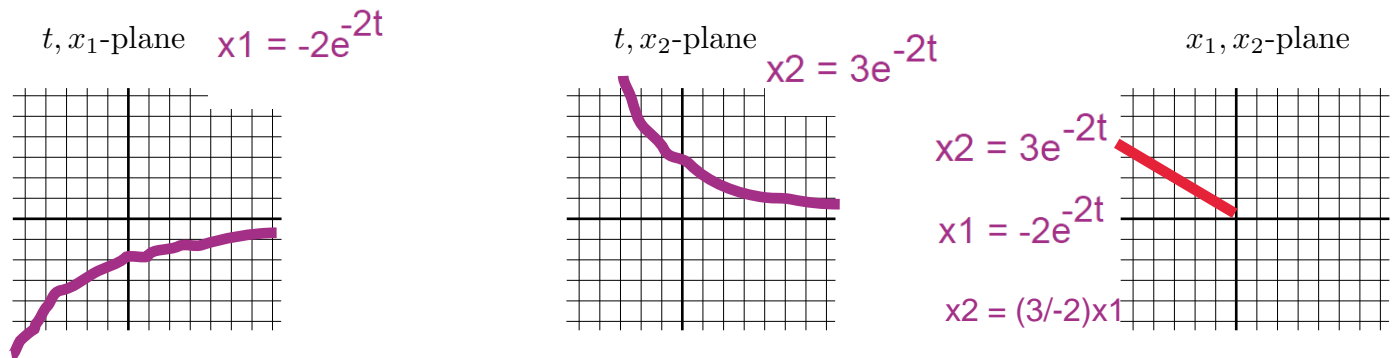$\int (t^4 y)' dt = \int t^4 dt$

$t^4 y = \frac{t^5}{5} + C$

$y = \frac{t}{5} + Ct^{-4}$


Answer: $y = \frac{t}{5} + Ct^{-4}$

2.) Give that the solution to $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} -2 \\ 3 \end{bmatrix} e^{-2t}$

Note c1 = 0 and c2 = 1 for this IVP.

[7] 2a.) Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ in the

Thus x1 = -2e^{-2t} and x2 = 3e^{-2t}

t, x_1-plane   x1 = -2e^{-2t}

t, x_2-plane

x2 = 3e^{-2t}

x_1, x_2-plane

x2 = 3e^{-2t}

x1 = -2e^{-2t}

x2 = (3/-2)x1

[3] 2b.) Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the
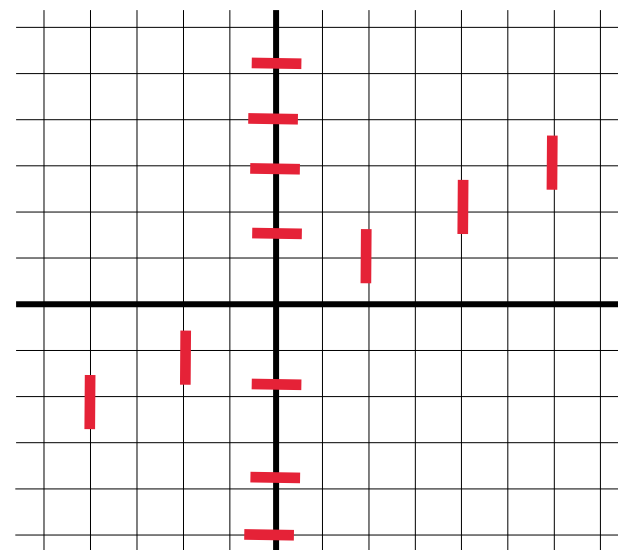
t, x_1-plane

t, x_2-plane
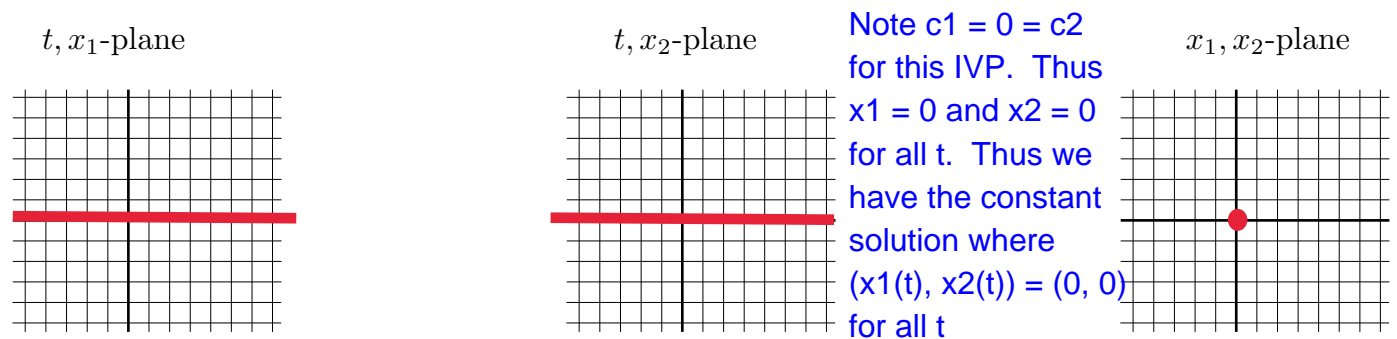
Note c1 = 0 = c2
for this IVP. Thus
x1 = 0 and x2 = 0
for all t. Thus we
have the constant
solution where
(x1(t), x2(t)) = (0, 0)
for all t

x_1, x_2-plane

[2] 2c.) The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

[3] 2d.) $\frac{dx_2}{dx_1} = \dfrac{\overline{3x1}}{1x1 + 2x2}$

[2] 2e.) Plot several direction vectors where the slope is 0 and where slope is vertical.

[10] 2f.) Graph several trajectories.