Quiz 2 SHOW ALL WORK
Sept 20, 2018

[10] 1.) Solve $ty' + (t+1)y = t$ (Compare to 2.1: 12)

$y' + \frac{(t+1)}{t}y = 1$

$u(t) = e^{\int \frac{(t+1)}{t}dt} = e^{\int (1+\frac{1}{t})dt} = e^{t+ln|t|} = e^t e^{ln|t|} = |t|e^t.$

Let $u(t) = te^t$

$te^t y' + e^t(t+1)y = te^t$

$(te^t y)' = te^t$ **Check:** $(te^t y)' = te^t y' + (e^t + te^t)y$

$\int (te^t y)' = \int te^t$

integration by parts:

$$\begin{array}{ll} u = t & dv = e^t \\ du = 1 & v = e^t \\ d^2u = 0 & \int v = e^t \end{array}$$

$te^t y = te^t - e^t + C$

Thus $y = 1 - t^{-1} + Ct^{-1}e^{-t}$

Answer: $\underline{y = 1 - t^{-1}e^t + Ct^{-1}e^{-t}}$

[10] 2.) Suppose ϕ_n is defined by sucessive approximation where $y' = \frac{2t}{y+1}$, $y(0) = 0$.

If $\phi_1(t) = t^2$, then $\phi_2(t) = \underline{ln|t^2+1|}$ (from 2.8)

$y' = f(t, y(t))$, $y(0) = 0$. Thus $y(t) = \int_0^t f(s, y(s))ds$.

$\phi_2(t) = \int_0^t f(s, \phi_1(s))ds = \int_0^t \frac{2s}{s^2+1} = ln|s^2+1||_0^t = ln|t^2+1| - ln|1| = ln|t^2+1|$