Quiz 5 SHOW ALL WORK
Nov 30, 2018

[13] 1.) Find all equilibrium solutions and classify them (stable, asymptotically stable, semi-stable, unstable and if system of DEs, node, saddle, spiral, center). For the non-linear system of DEs, state all possibilities for type of equilibrium solution.

1a.) $y' = (y-3)(y-5)^8$ $y = 3$ is unstable, $y = 5$ is semi-stable.

1b.) $x' = y - 1,\ \ y' = (x-3)y$

If $y - 1 = 0$, then $y = 1$

If $y = 1$, then $(x-1)y = x - 3 = 0$. Thus $x = 3$.

Jacobian matrix: $\begin{bmatrix} 0 & 1 \\ y & x-3 \end{bmatrix}$

For $(x, y) = (1, 3)$, Jacobian matrix is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

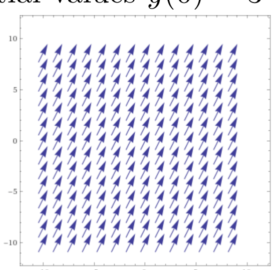
$$\begin{vmatrix} -r & 1 \\ 1 & -r \end{vmatrix} = r^2 - 1 = (r-1)(r+1) = 0.$$

Thus $r = -1.1$, i.e, one positive and one negative eigenvalue.

Thus $(x(t), y(t)) = (1, 3)$ is an unstable saddle.

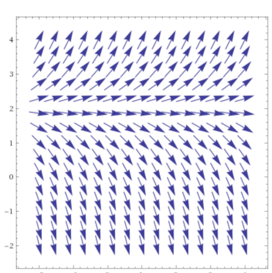
[7] 2.) The slope field for a first order differential equation is shown below. In addition to determining and classifying all equilibrium solutions (if any), also draw the trajectories satisfying the initial values $y(0) = 3$ and $y(1) = 0$.

2a.)



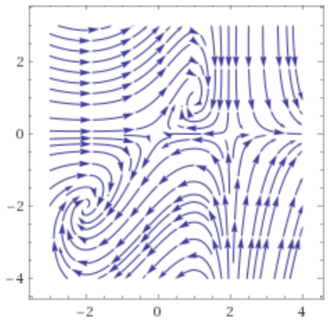
No equilibrium solution.

2b.)



Unstable quilibrium solution: $y = 2$.

[8] 3.) The stream plot in the x_1-x_2 plane for a system of two first order differential equations is shown below. In addition to determining and classifying the **4** equilibrium solutions, also draw the trajectory satisfying the initial value $(x_1(0), x_2(0)) = (0, -2)$. Also describe the basins of attraction for each asymptotically stable equilibrium solutions.



$(x_1(t), x_2(t)) = (0, 0)$ is an unstable saddle.

$(x_1(t), x_2(t)) = (2, 0)$ is an unstable saddle.

$(x_1(t), x_2(t)) = (1, 1)$ is an asymptotically stable spiral.

basin of attraction: $x_1 < 2$ and $x_2 > 0$.

$(x_1(t), x_2(t)) = (-2, -2)$ is an asymptotically stable spiral.

basin of attraction: $x_1 < 2$ and $x_2 < 0$.

[5] 4.) Use Picard's iteration method to find a degree 3 polynomial approximation for the solution to the initial value problem, $y' = y + 6t^2$, $y(0) = 0$. Start with $\phi_0(t) = 0$.

$\phi_1(t) = \int_0^t f(s, \phi_0(s))ds = \int_0^t (\phi_0(s) + 6s^2)ds = \int_0^t (6s^2)ds = \int_0^t 2s^3|_0^2 = 2t^3$

Approximation: $\underline{\quad y = 2t^3 \quad}$

[7] 5.) Using power series to find a degree 3 polynomial approximation for the general solution to $y' - y = 6x^2$ for x near 0

$y = \Sigma_{n=0}^{\infty} a_n x^n$, $y' = \Sigma_{n=1}^{\infty} a_n n x^{n-1}$,

$\Sigma_{n=1}^{\infty} a_n(n)x^{n-1} - \Sigma_{n=0}^{\infty} a_n x^n = 6x^2$

$\Sigma_{n=0}^{\infty} a_{n+1}(n+1)x^n - \Sigma_{n=0}^{\infty} a_n x^n = 6x^2$

$\Sigma_{n=0}^{\infty} [a_{n+1}(n+1) - a_n]x^n = 6x^2$

$a_{n+1}(n+1) - a_n = 0$ for $n = 0, 1$ and $n > 2$

$a_{n+1} = \frac{a_n}{n+1}$ for $n = 0, 1$ and $n > 2$

For $n = 0$: $\quad a_1 = \frac{a_0}{1}$

For $n = 1$: $\quad a_2 = \frac{a_1}{2} = \frac{a_0}{2}$

For $n = 2$: $\quad a_3(3) - a_2]x^2 = 6x^2$ implies $3a_3 - a_2 = 6$ and thus $a_3 = \frac{a_2+6}{3} = \frac{a_2}{3} + 2 = \frac{a_0}{6} + 2$

Answer: $\underline{\quad y = a_0 + a_0 x + \frac{a_0}{2}x^2 + (\frac{a_0}{6} + 2)x^3 \quad}$

Note for this particular example, the approximation for the IVP solutions are the same for both 4 and 5. This is NOT usually the case. Different methods to approximate solutions will generaly give different approximations, but in the limit they should be the same.