Math 3600 Differential Equations Exam #2

Oct 26, 2018 SHOW ALL STEPS

1.) Circle T for true and F for false.

[4] 1a.) Suppose $f(x) = \Sigma a_n(x-3)^n$ has a radius of convergence $= r$ about 3. Then we can define the domain of f to be $(3-r, 3+r)$.

T F

[4] 1b.) If $b^2 - 4ac < 0$, then the solution to the initial value problem $ay'' + by' + cy = 0$, $y(0) = 2$, $y'(0) = 1$ is a complex valued function.

T F

[4] 1c.) If $b^2 - 4ac < 0$, then the solution to the characteristic equation $ar^2 + br + c = 0$ is complex valued.

T F

[4] 1d.) $D(f) = f'$ is a linear function.

T F

[4] 1e.) There is a unique solution to the differential equation $ay''+by'+cy = g(t), y(0) = 1, y(1) = 0$

T F

[7] 2.) The eigenvalues of $\begin{pmatrix} 3 & -2 \\ 1 & 5 \end{pmatrix}$ are ____

[7] 3.) Suppose $A\begin{bmatrix} 4 \\ 12 \end{bmatrix} = \begin{bmatrix} -3 \\ 11 \end{bmatrix}$, $A\begin{bmatrix} 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 21 \end{bmatrix}$, $A\begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 31 \end{bmatrix}$, $A\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$

State the 2 eigenvalues of A:

State 5 eigenvectors of A:

[20] 4.) Using power series, find a degree 5 polynomial approximation for the solution to $y'' - y = 4x$ for x near 0

Approximation: ________________________________

[22] 5.) Solve $y'' - y = e^t + 2, \quad y(0) = 1,\, y'(0) = 2$

Solution: ____

[24] 6.) Solve **two** of the following (from this page and the next page). If you solve all 4, I will grade your best 2 and will give 1 (or 2) points extra credit for 3 (or 4) correct problems):

6a.) If $y = \psi(t)$ is a solution to $py'' + qy' + ry = g(t)$, show that $y = 2\psi(t)$ is a solution to $py'' + qy' + ry = 2g(t)$. Hint use linearity OR plug in.

6b.) Use your work in problem 5 to solve $y'' - y = 3e^t + 10$ for the general solution.

6c.) Given a_0, a_1 and $a_{n+2} = 2a_{n+1} - a_n$, determine a_n in terms of a_0 and a_1.

6d.) Use the ratio test to determine the radius of convergence for the power series $\Sigma_{n=0}^{\infty} \frac{3^n}{2n-1} x^n$. For what values of x does this series converge?