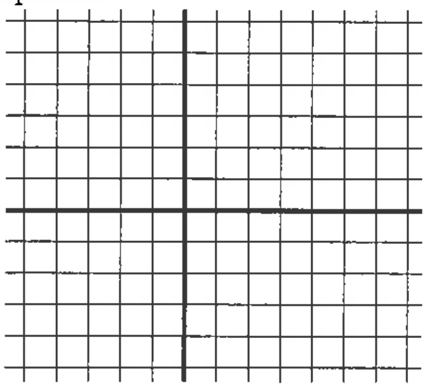
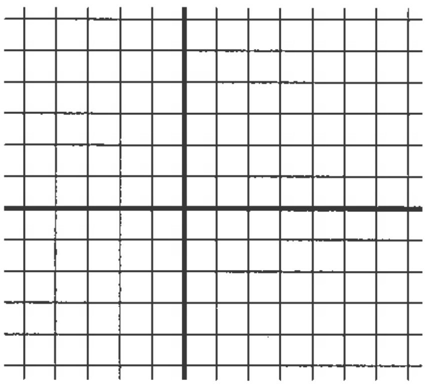
8.1 supplemental HW

1.) For each of the following differential equations (i) draw its direction field; (ii) sketch the solution of the direction field that passes through the point (-2, 1); (iii) state the general solution to the differential equation.

a.) $y' = 0$



b.) $y' = -1$



$y = t + 1$

2.) Circle a solution to the differential equation whose direction field is given below:

~~A) $y = t^2$~~

B) $y = \frac{1}{2}t + 1$

~~C) $y = e^t$~~

D) $y = t + 1$

~~E) $y = -2e^t$~~

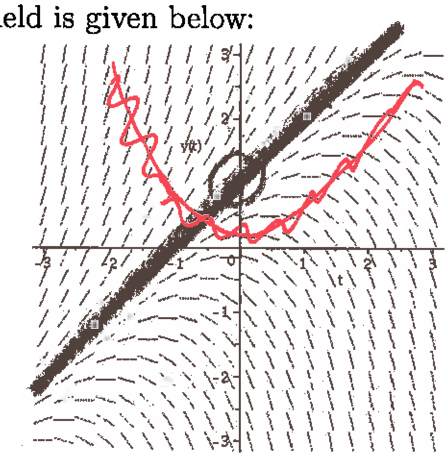
F) $y = 2t + 1$

~~G) $y = ln(t)$~~

~~H) $y = 0$~~

~~I) $y = sin(t)$~~

~~J) $y = cos(t)$~~



3.) Circle the differential equation whose direction field is given below:

A) $y' = t^2$

B) $y' = \frac{1}{2}t + 1$

C) $y' = e^t$

D) $y' = t + 1$

E) $y' = -2e^t$

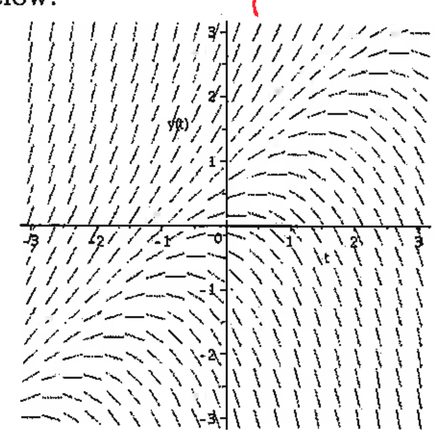
F) $y' = y - t$

G) $y' = ln(t)$

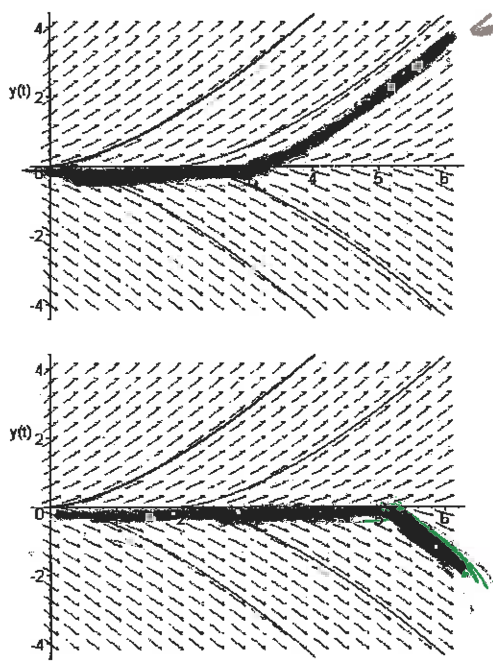
H) $y' = 0$

I) $y' = sin(t)$

J) $y' = cos(t)$



$y' = y^{1/3}$



$\leftarrow y(3) = 0$

Figure 2.4.1 from *Elementary Differential Equations and Boundary Value Problems*, Eighth Edition by William E. Boyce and Richard C. DiPrima

Note IVP, $y' = y^{\frac{1}{3}}$, $y(x_0) = 0$ has an infinite number of solutions,

while IVP, $y' = y^{\frac{1}{3}}$, $y(x_0) = y_0$ where $y_0 \neq 0$ has a unique solution.

Initial Value Problem: $y(t_0) = y_0$
Use initial value to solve for C.

Section 2.4: Existence and Uniqueness.

In general, for $y' = f(t, y)$, $y(t_0) = y_0$, solution may or may not exist and solution may or may not be unique.

$\boxed{\textbf{Example Non-unique: } y' = y^{\frac{1}{3}}}$

$y = 0$ is a solution to $y' = y^{\frac{1}{3}}$ since $y' = 0 = 0^{\frac{1}{3}} = y^{\frac{1}{3}}$

Suppose $y \neq 0$. Then $\frac{dy}{dx} = y^{\frac{1}{3}}$ implies $y^{-\frac{1}{3}}dy = dx$

$\int y^{-\frac{1}{3}}dy = \int dx$ implies $\frac{3}{2}y^{\frac{2}{3}} = x + C$

$y^{\frac{2}{3}} = \frac{2}{3}x + C$ implies $y = \pm\sqrt{(\frac{2}{3}x + C)^3}$

Suppose $y(3) = 0$. Then $0 = \pm\sqrt{(2 + C)^3} \Rightarrow C = -2.$[redacted]

to IVP

The IVP, $y' = y^{\frac{1}{3}}$, $y(3) = 0$, has an infinite # of sol'ns[redacted]

$x \geq 3$

including: $y = 0$, $y = \sqrt{(\frac{2}{3}x - 2)^3}$, $y = -\sqrt{(\frac{2}{3}x - 2)^3}$

$0 \quad x \leq 3$

slope field

Browse Examples   Surprise Me

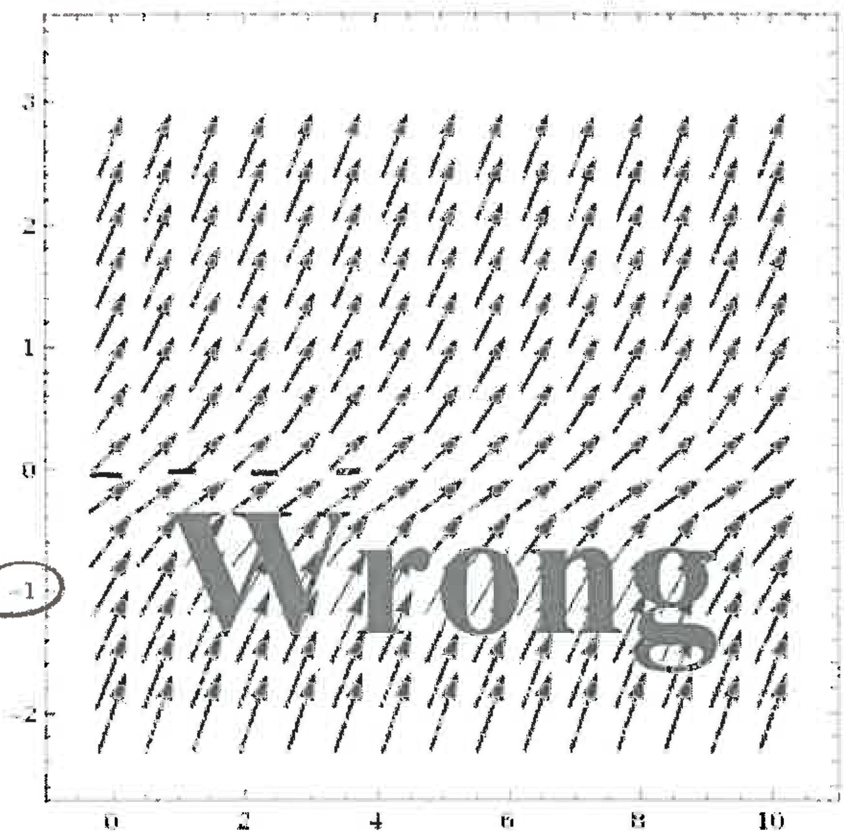
Assuming "slope field" refers to a computation | Use as referring to a mathematical definition instead

- vector field: {1, y^(1/3)}/sqrt(y^(2/
- variable 1: x
- lower limit 1: 0
- upper limit 1: 10
- variable 2: y
- lower limit 2: -2
- upper limit 2: 3

Input:

$$\text{VectorPlot}\left[\frac{\{1, \sqrt[3]{y}\}}{\sqrt{y^{2/3}+1}}, \{x, 0, 10\}, \{y, -2, 3\}\right]$$

-1

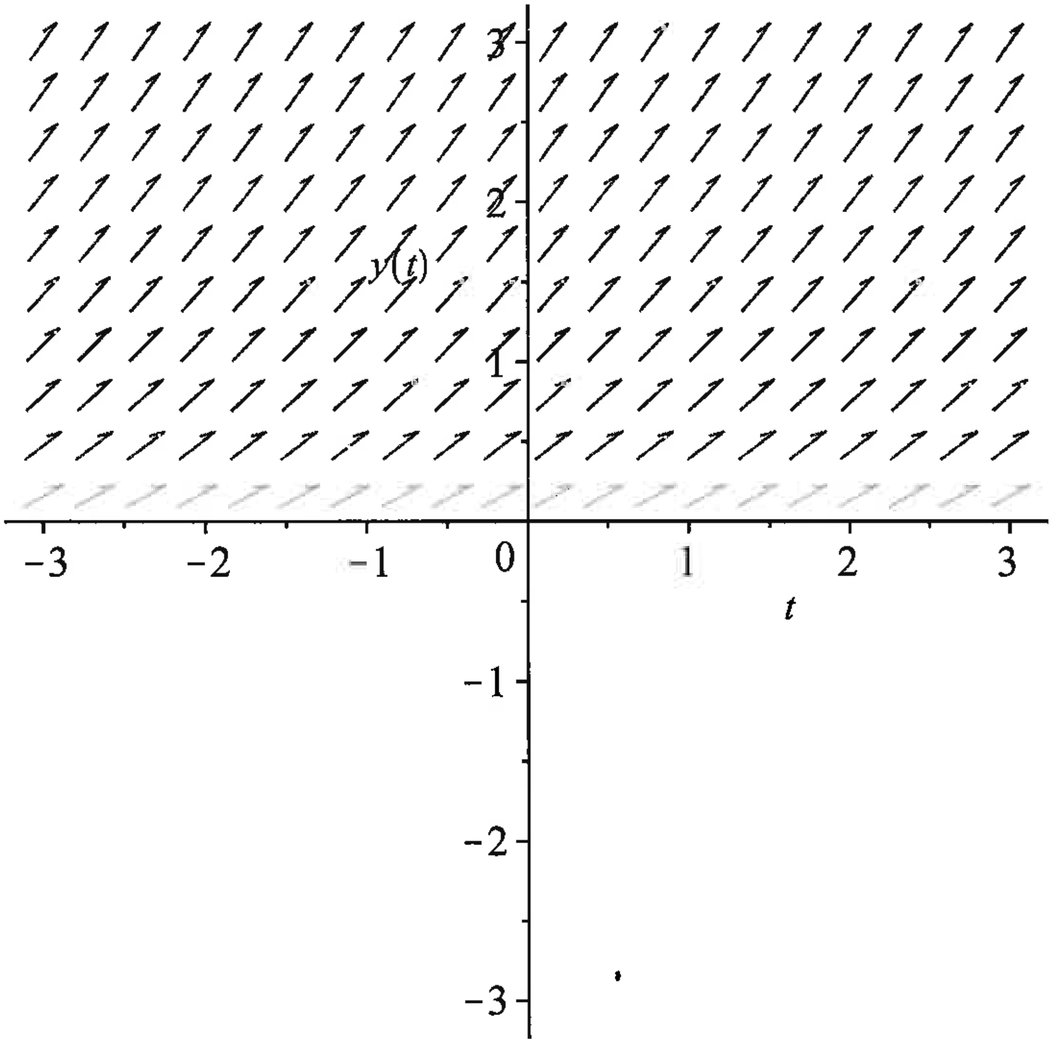


0 - 10

> with(DEtools) :

>

> $dfieldplot\left(\frac{d}{dt}\,y(t) = y(t)^{\left(\frac{1}{3}\right)}, y(t), t=-3\,..3, y=-3\,..3, color=y(t)\right)$



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IVP: Does C has unique soln when plugging in initial value $y(t_0) = y_0$

Examples: No solution:

Ex 1: $y' = y' + 1$

Ex 2: $(y')^2 = -1$

Ex 3 (IVP): $\frac{dy}{dx} = y(1 + \frac{1}{x})$, $y(0) = 1$

$\int \frac{dy}{y} = \int (1 + \frac{1}{x}) dx$ implies $ln|y| = x + ln|x| + C$

$|y| = e^{x + ln|x| + C} = e^x e^{ln|x|} e^C = C|x|e^x = Cxe^x$

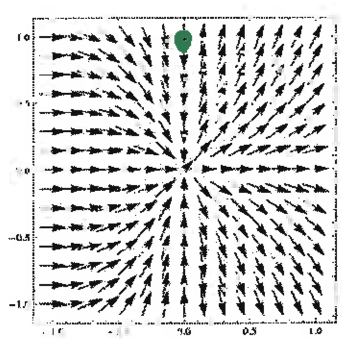
$y = \pm Cxe^x$ implies $y = Cxe^x$

$y(0) = 1$: $1 = C(0)e^0 = 0$ implies

IVP $\frac{dy}{dx} = y(1 + \frac{1}{x})$, $y(0) = 1$ has no solution.
to IVP

http://www.wolframalpha.com

slope field: $\{1, y(1+1/x)\}/sqrt(1+y\wedge2(1+1/x)\wedge2)$



Special cases:

Calc 1

Suppose f is cont. on (a,b) and the point $t_0 \in (a,b)$,
Solve IVP: $\frac{dy}{dt} = f(t)$, $y(t_0) = y_0$

$$dy = f(t)dt$$

$$\int dy = \int f(t)dt$$

$y = F(t) + C$ where F is any anti-derivative of F.

Initial Value Problem (IVP): $y(t_0) = y_0$

$$y_0 = F(t_0) + C \text{ implies } C = y_0 - F(t_0)$$

Hence unique solution (if domain connected) to IVP:

$$y = F(t) + y_0 - F(t_0)$$

First order linear differential equation:

Thm 2.4.1: If p and g are continuous on (a,b) and the point $t_0 \in (a,b)$, then there exists a unique function $y = \phi(t)$ defined on (a,b) that satisfies the following initial value problem:

$$y' + p(t)y = g(t), \quad y(t_0) = y_0.$$