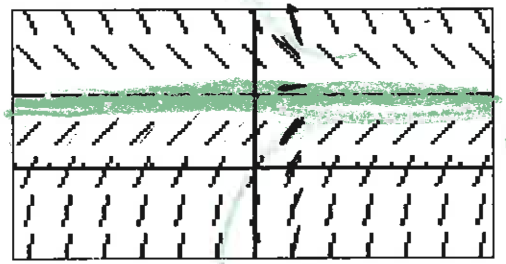
Match the slope fields with their differential equations.

$y \to 2$ as $t \to +\infty$

(A)



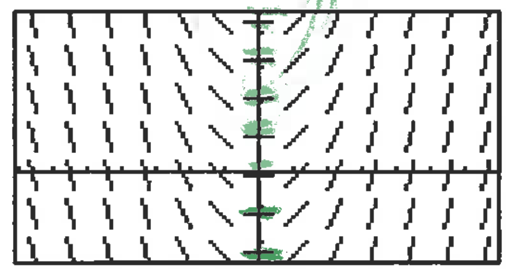
Equilibrium

$y = 2$

$y' = f(y)$

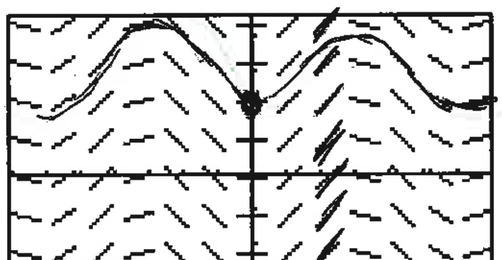
$t \to \infty \Rightarrow y \to \infty$

(B)



$y' = f(x)$

(C)



$y' = f(x)$

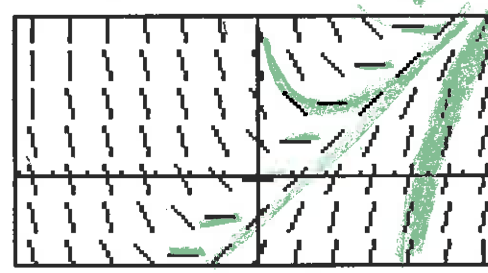
$y' = 0$

$0 = 2 - y$

$2 = y$

$t \to \infty \Rightarrow y \to \infty$

(D)



$y' = f(x, y)$

7. $\dfrac{dy}{dx} = \sin x$ C

8. $\dfrac{dy}{dx} = x - y$ D

9. $\dfrac{dy}{dx} = 2 - y$ A

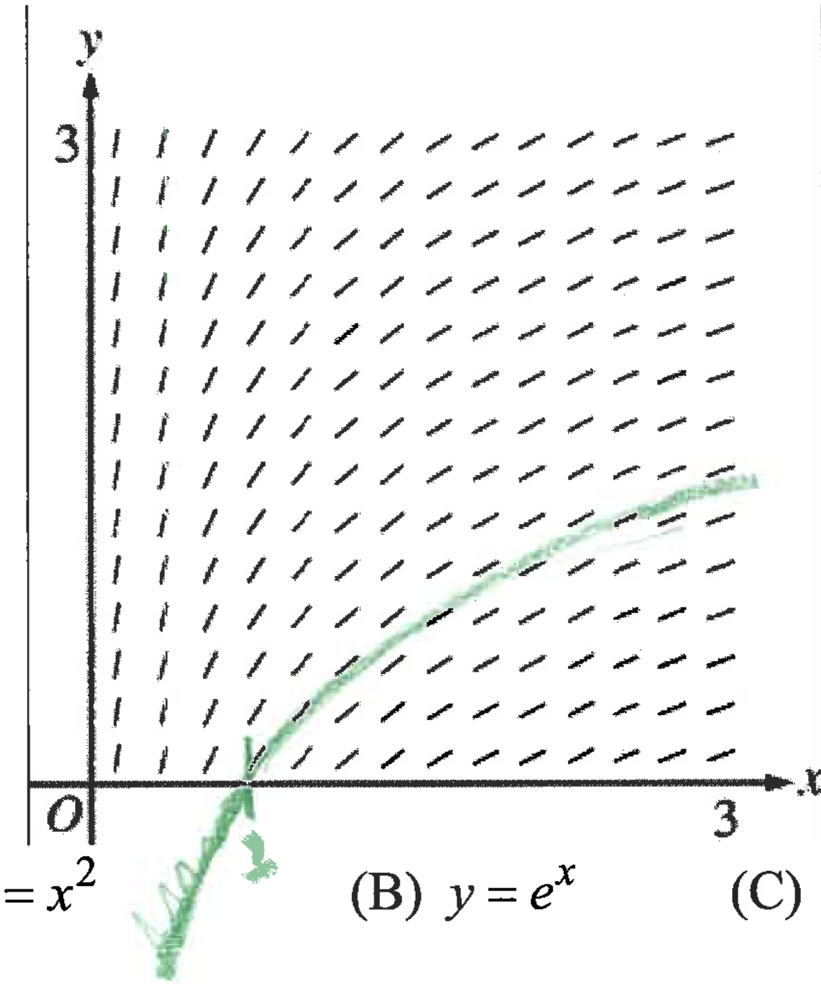
10. $\dfrac{dy}{dx} = x$ B

$\int dy = \int \sin x \, dx \Rightarrow y = -\cos x + C$

From the May 2008 *AP Calculus Course Description:*

From: http://apcentral.collegeboard.com/apc/public/repository/ap08_calculus_slopefields_worksheet.pdf

15.



(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$

The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$

http://www.wolframalpha.com

slope field

Web Apps   Examples   Random

Assuming "slope field" refers to a computation | Use as referring to a mathematical definition instead

{1, (ln(x) + y)}/sqrt(1 + (ln(x) + y)^2)

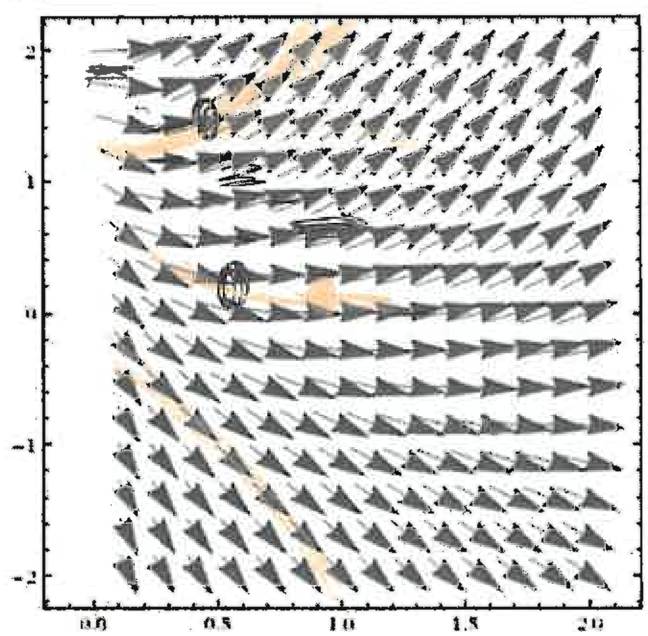
vector field	{1, (ln(x) + y)}/sqrt(1
variable 1	x
lower limit 1	0
upper limit 1	2
variable 2	y
lower limit 2	-2
upper limit 2	2

Input

$$\text{VectorPlot}\left[\frac{\{1, \log(x) + y\}}{\sqrt{1 + (\log(x) + y)^2}}, \{x, 0, 2\}, \{y, -2, 2\}\right]$$

log(x) is the natural logarithm

Result



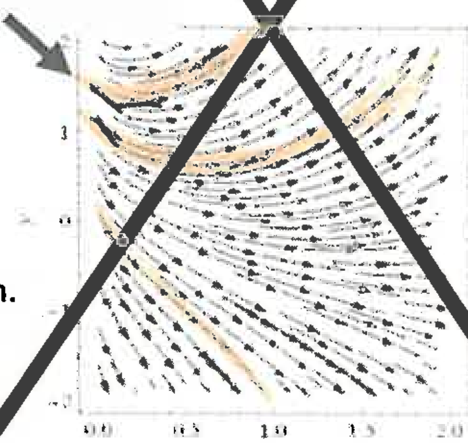
StreamPlot[{1, (ln(x) + y)}, {x, 0, 2}, {y, -2, 2}]

Input interpretation

stream plot	1 log(x) + y	x = 0 to 2 y = −2 to 2

Do NOT curve

your slope lines



Slope lines are small portions of lines tangent to a solution.
Thus slope lines must be straight. They cannot be curved.

Arrows are optional

Integration by parts:

Derivative of a product: $(uv)' = uv' + vu'$

$$uv' = (uv)' - vu'$$

$$\int uv' = \int (uv)' - \int vu'$$

$$\int uv' = (uv) - \int vu'$$

Example: $\int e^{2x} sin(3x)$

Let $u = sin(3x)$, $dv = e^{2x}$

then $du = 3cos(3x)$, $v = \frac{1}{2}e^{2x}$

then $d^2u = -9sin(3x)$, $\int v = \frac{1}{4}e^{2x}$

$\int e^{2x} sin(3x) = \frac{1}{2} sin(3x)e^{2x} - \int \frac{3}{2} e^{2x} cos(3x)$ ← int by parts

$= \frac{1}{2} sin(3x)e^{2x} + [\frac{3}{4} cos(3x)e^{2x} + \int \frac{-9}{4} sin(3x)e^{2x}$ ← int by

$\int e^{2x} sin(3x) = \frac{1}{2} sin(3x)e^{2x} - \frac{3}{4} cos(3x)e^{2x} - \frac{9}{4} \int sin(3x)e^{2x}$

$\frac{13}{4} \int e^{2x} sin(3x) = \frac{1}{2} sin(3x)e^{2x} - \frac{3}{4} cos(3x)e^{2x}$

$\int e^{2x} sin(3x) = \frac{4}{13} [\frac{1}{2} sin(3x)e^{2x} - \frac{3}{4} cos(3x)e^{2x}]$

Optional Exercise: Calculate $\int e^x cos(2x)$

Calulus pre-requisites you must know.

Derivative = slope of tangent line = rate.

Integral = area between curve and x-axis (where area can be negative).

The Fundamental Theorem of Calculus: Suppose f continuous on $[a, b]$.

1.) If $G(x) = \int_a^x f(t)dt$, then $G'(x) = f(x)$.

I.e., $\frac{d}{dx}[\int_a^x f(t)dt] = f(x)$.

2.) $\int_a^b f(t)dt = F(b) - F(a)$ where F is any antiderivative of f, that is $F' = f$.

Integration Pre-requisites:

* Integration by substitution
* Integration by parts
* Integration by partial fractions

~~Note: Partial fractions are also used in ch 6 for a different application.~~

Suppose f is cont. on (a, b) and the point $t_0 \in (a, b)$,

Solve IVP: $\frac{dy}{dt} = f(t), \quad y(t_0) = y_0$

Calc 1 problem

separate variables $\rightarrow$ $dy = f(t)dt$

$$\int dy = \int f(t)dt$$

$y = F(t) + C$ where F is any anti-derivative of F.

Initial Value Problem (IVP): $y(t_0) = y_0$ (t_0, y_0)

$$y_0 = F(t_0) + C \text{ implies } C = y_0 - F(t_0)$$

Hence unique solution (if domain connected) to IVP:

$$y = F(t) + y_0 - F(t_0)$$

$$\boxed{\frac{dy}{dt} = ry + k}$$

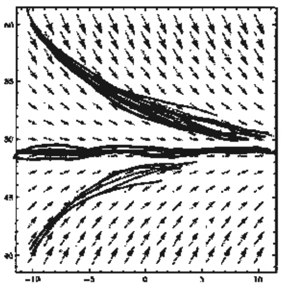
1.1: Examples of differentiable equation:

1.) $F = ma = m\frac{dv}{dt} = mg - \gamma v$

$$\boxed{\frac{dv}{dt} = g - \frac{\gamma}{m} v}$$

$$0 = g - \frac{r}{m} v$$

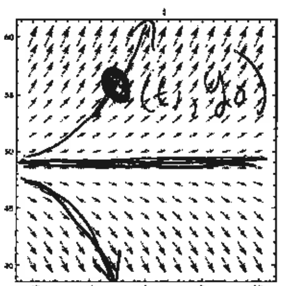
$$\Rightarrow v = \frac{mg}{\gamma}$$



2.) Mouse population increases at a rate proportional[redacted]
to the current population:

$r > 0$

More general model : $\frac{dp}{dt} = rp - k$
where $p(t)$ = mouse population at time t,
r = growth rate or rate constant,
k = predation rate = # mice killed per unit time.



3.) Continuous compounding $\frac{dS}{dt} = rS + k$
where $S(t)$ = amount of money at time t,
r = interest rate,
k = constant deposit rate

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direction field = slope field = graph of $\frac{dy}{dt}$ in t, y-plane.

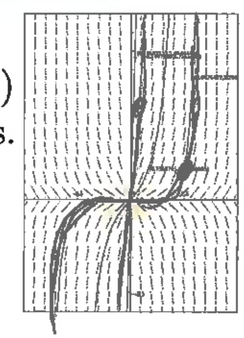
*** can use slope field to determine behavior of y including as $t \to \pm\infty$

*** Equilibrium Solution = constant solution

Most differential equations do not have an equilibrium solution.

$y = C$

$y' = 0$



Initial value: A chosen point (t_0, y_0) through which a solution must pass. I.e., (t_0, y_0) lies on the graph of the solution that satisfies this initial value.

Initial value problem (IVP): A differential equation where initial value is specified.

An initial value problem can have 0, 1, or multiple equilibrium solutions.

************Existence of a solution**************

************Uniqueness of solution*************

1.3:

ODE (ordinary differential equation): single independent variable

Ex: $\frac{dy}{dt} = ay + b$

PDE (partial differential equation): several independent variables

Ex: $\frac{\partial xy}{\partial x} = \frac{\partial xy}{\partial y}$

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order of differential eq'n: order of highest derivative

example of order n: $y^{(n)} = f(t, y, ..., y^{(n-1)})$

Linear vs Non-linear

Linear: $a_0 y^{(n)} + ... + a_{n-1}y' + a_n y = g(t)$

where a_i's are functions of t

Note for this linear equation, the left hand side is a linear combination of the derivatives of y (denoted by $y^{(k)}, k = 0, ..., n$) where the coefficient of $y^{(k)}$ is a function of t (denoted $a_k(t)$).

Linear: $a_0(t)y^{(n)} + ... + a_{n-1}(t)y' + a_n(t)y = g(t)$

Determine if linear or non-linear:

Ex: $ty'' - t^3y' - 3y = sin(t)$

Ex: $2y'' - 3y' - 3y^2 = 0$

Show that for some value of r, $y = e^{rt}$ is a soln to the 1^{rst} order linear homogeneous equation $2y' - 6y = 0$.

To show something is a solution, plug it in:

$y = e^{rt}$ implies $y' = re^{rt}$. Plug into $2y' - 6y = 0$:

$2re^{rt} + 6e^{rt} = 0$ implies $2r - 6 = 0$ implies $r = 3$

Thus $y = e^{3t}$ is a solution to $2y' - 6y = 0$.

Show $y = Ce^{3t}$ is a solution to $2y' - 6y = 0$.

$$\begin{aligned} 2y' - 6y = 2(Ce^{3t})' - 6(Ce^{3t}) &= 2C(e^{3t})' - 6C(e^{3t}) \\ &= C[2(e^{3t})' - 6(e^{3t})] = C(0) = 0. \end{aligned}$$