2.7 Euler Method

$\boxed{\text{Use tangent lines to approximate sol'n to IVP}}$

Ex IVP: $y' = y^2$, $y(2) = 1$

Sidenote: by § 2.4 theorem 2.4.2, $f(y) = y^2$ & $\frac{\partial f}{\partial y} = 2y$ are continuous

$\Rightarrow$ sol'n to IVP is exists & is unique on some interval containg $t_0 = 2$

§ 2.2 separable, not linear

Solve $\frac{dy}{dt} = y^2$, $y(2) = 1$

$$\int y^{-2}\,dy = \int dt \Rightarrow -y^{-1} = t + C$$

$$\Rightarrow y = \frac{1}{-t+C} \quad \& \quad 1 = \frac{1}{-2+C} \Rightarrow C = 3$$

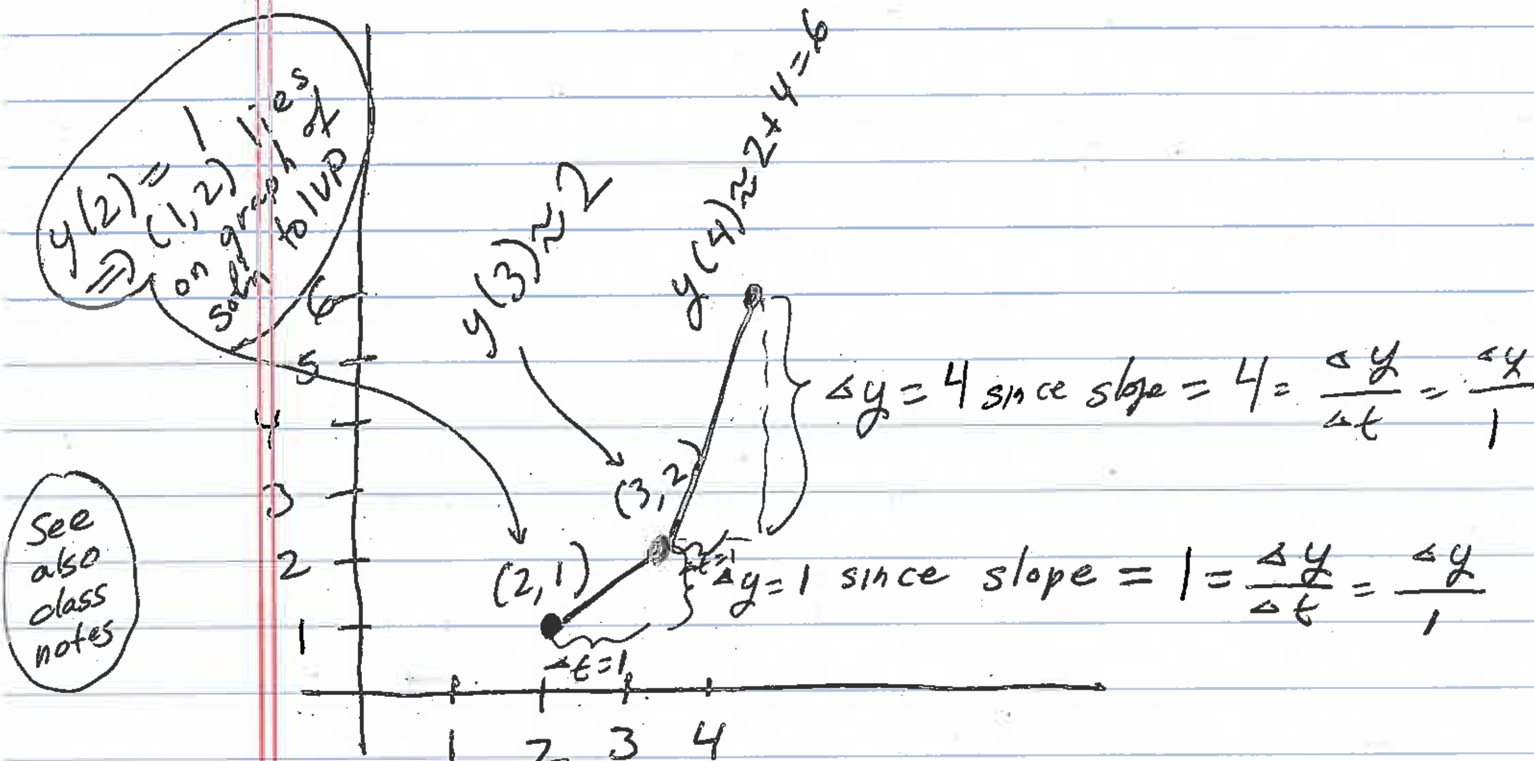
$$y = \frac{1}{3-t}$$

But can't always solve IVP so

§2.7: use multiple tangent lines to approx soln

Ex1: IVP: $y' = y^2$, $y(2) = 1$

Ex 1a: Let $\Delta t = 1$



slope at (2,1): $y' = y'(t,y) = y^2$

$y'(2,1) = 1^2 = 1$

$y(2) = 1$

$y(2+\Delta t) = y(2+1) = y(3) \approx y(2) + \Delta y$

$= y(2) + \text{slope} \cdot \Delta t = 1 + (1)(1) = 2$

slope at (3,2): $y'(3,2) = 2^2 = 4$

t	exact $y = \frac{1}{3-t}$	approx y
2	1	1
3	not defined	2
4	$\frac{1}{3-4} = -1$	6

bad approx
Δt too large

Better approx: choose smaller Δt

See class notes + chalk board for

Ex 1b: $\Delta t = 0.1$

Ex 2: IVP $y' = t + 2y$, $y(0) = 0$

Let $\Delta t = 0.1$

§2.4: $f(t,y) = t+2y$
$\frac{\partial f}{\partial y}(t+2y) = 2$
$\left.\right\}$ cont $\forall t,y$ $\Rightarrow$ unique soln in some interval about $t=0$

§2.1: Linear, not separable

$y' - 2y = t$ $\qquad u(t) = e^{\int -2dt} = e^{-2t}$

$e^{-2t}y' - 2e^{-2t}y = te^{-2t}$

$u = t \quad dv = e^{-2t}$
$du = dt \quad v = \frac{e^{-2t}}{-2}$

$e^{-2t}y = \int (e^{-2t}y)' dt = \int te^{-2t} = \frac{-te^{-2t}}{2} - \frac{e^{-2t}}{4} + C$

$y = -\frac{t}{2} - \frac{1}{4} + Ce^{2t}$

§2.7: Approx soln using multiple tangent lines

$y' = t + 2y$, $y(0) = 0$

$y'(0,0) = 0 + 2(0) = 0 \Leftarrow$ slope 0

slope $= \frac{\Delta y}{\Delta t}$

Given $\Delta t = 0.1$

t	approx y
0	0
0.1	0
0.2	0.01
0.3	0.032

$0 + \text{slope}\,\Delta t = 0 + 0(0.1)$

$y'(0.1, 0) = 0.1 + 2(0) = 0.1$

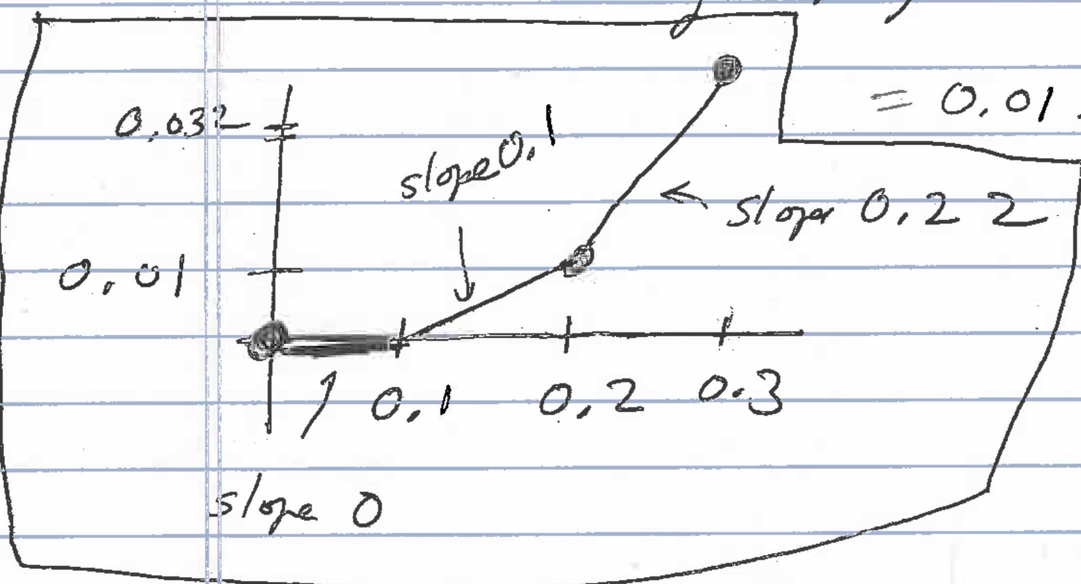
$y(0.2) = y(0.1) + \Delta y = 0 + (0.1)(0.1)$

$y'(0.2, 0.01) = 0.2 + 2(0.01)$

$= 0.22$

$y(0.3) = y(0.2) + \Delta y = 0.01 + (0.22)(0.1)$

$= 0.01 + 0.022 = 0.032$



eqn of tangent lines

at $t = 0$: $y = 0$

at $t = 0.1$: slope $= \frac{\Delta y}{\Delta t} = \frac{\text{rise}}{\text{run}} \Rightarrow 0.1 = \frac{y - 0}{t - 0.1}$

$\Rightarrow \boxed{y = 0.1t - 0.01}$

at $t = 0.2$: $0.22 = \frac{y - 0.01}{t - 0.2}$

$y = 0.22t - 0.044 + 0.01$

Approx soln using multiple tangent lines

$$y = \begin{cases} 0 & 0 \le t \le 0.1 \\ 0.1t - 0.01 & 0.1 \le t \le 0.2 \\ 0.22t - 0.34 & 0.2 \le t \le 0.3 \\ \vdots & \end{cases}$$