Quiz 6 Math 2560 Sect 81 Show your work
Dec 2, 2016

[3] 1.) Circle T for True and F for false:

1a.) If ϕ is a solution to a first order **linear homogeneous** differential equation, then $c\phi$ is also a solution to this equation. T

1b.) If ϕ is a solution to a first order linear differential equation, then $c\phi$ is also a solution to this equation. F

[4] 2a.) Solve: $\mathbf{x}' = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix} \mathbf{x}$

Find eigenvalues:

$$\begin{vmatrix} 1-r & 5 \\ 2 & 3-r \end{vmatrix} = (1-r)(3-r) - 10 = r^2 - 4r + 3 - 10 = r^2 - 4r - 7 = 0$$

$$r = \frac{4 \pm \sqrt{16-4(-7)}}{2} = \frac{4 \pm \sqrt{4(4+7)}}{2} = \frac{4 \pm 2\sqrt{11}}{2} = 2 \pm \sqrt{11}.$$

$2 + \sqrt{11} > 0$ and $2 - \sqrt{11} < 0$. Thus the critical point $\mathbf{x} = \mathbf{0}$ is an unstable saddle point.

Find eigenvectors:

$$\begin{bmatrix} 1-(2\pm\sqrt{11}) & 5 \\ 2 & 3-(2\pm\sqrt{11}) \end{bmatrix} = \begin{bmatrix} -1 \mp \sqrt{11} & 5 \\ 2 & 1 \mp \sqrt{11} \end{bmatrix}$$

$$\begin{bmatrix} -1 \mp \sqrt{11} & 5 \\ 2 & 1 \mp \sqrt{11} \end{bmatrix} \begin{bmatrix} 5 \\ 1 \pm \sqrt{11} \end{bmatrix} = \begin{bmatrix} 5(-1 \mp \sqrt{11}) + 5(1 \pm \sqrt{11}) \\ 10 + (1 \mp \sqrt{11})(1 \pm \sqrt{11}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus $\begin{bmatrix} 5 \\ 1 \pm \sqrt{11} \end{bmatrix}$ is an eigenvector with eigenvalue $2 \pm \sqrt{11}$.

Answer: $$\mathbf{x} = c_1 \begin{bmatrix} 5 \\ 1+\sqrt{11} \end{bmatrix} e^{(2+\sqrt{11})t} + c_2 \begin{bmatrix} 5 \\ 1-\sqrt{11} \end{bmatrix} e^{(2-\sqrt{11})t}$$

[2] 2b.) Determine if the critical point is stable, asymptotically stable, or unstable.
unstable

[1] 2c.) Draw the direction field and sketch a few trajectories.

Graph $x_2 = \frac{1+\sqrt{11}}{5} x_1$ and $x_2 = \frac{1-\sqrt{11}}{5} x_1$ for $x_1 > 0$ and $x_1 < 0$ along with some other trajectories.